$p(\bar{x}, \bar{y})$ $\quad$ $p(\bar{y}|\bar{x})$

explicit density

Naive Bayes:

$$p(\bar{x}, y) = p(y) \prod_i p(x_i|y)$$

Log. regr.:

$$p(y_i|\bar{x}) = \frac{1}{z(\bar{x})} e^{\bar{w}_i^T \bar{x}}$$

Explicit density

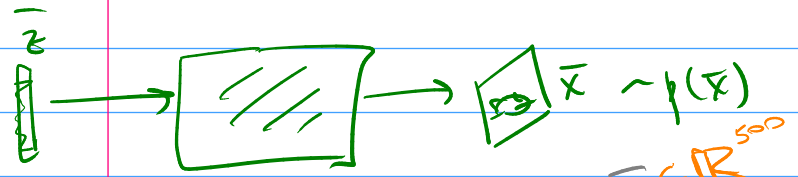
$$p(\bar{x}) = p(x_1)\, p(x_2|x_1) \ldots p(x_k|x_1 \ldots x_{k-1}) \ldots p(x_n|x_1 \ldots x_{n-1})$$

HMM: $p(x_k|x_1 \ldots x_{k-1}) = p(x_k|x_{k-1})$

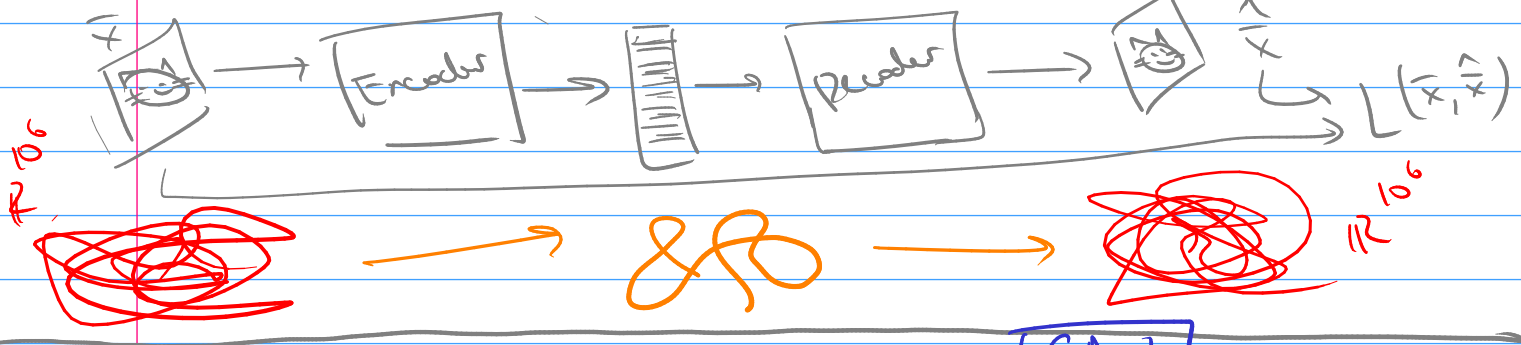
RNN: $p(x_k|x_1 \ldots x_{k-1}) = p(x_k|h_k)$

$h_k = f(x_{k-1}, h_{k-1})$

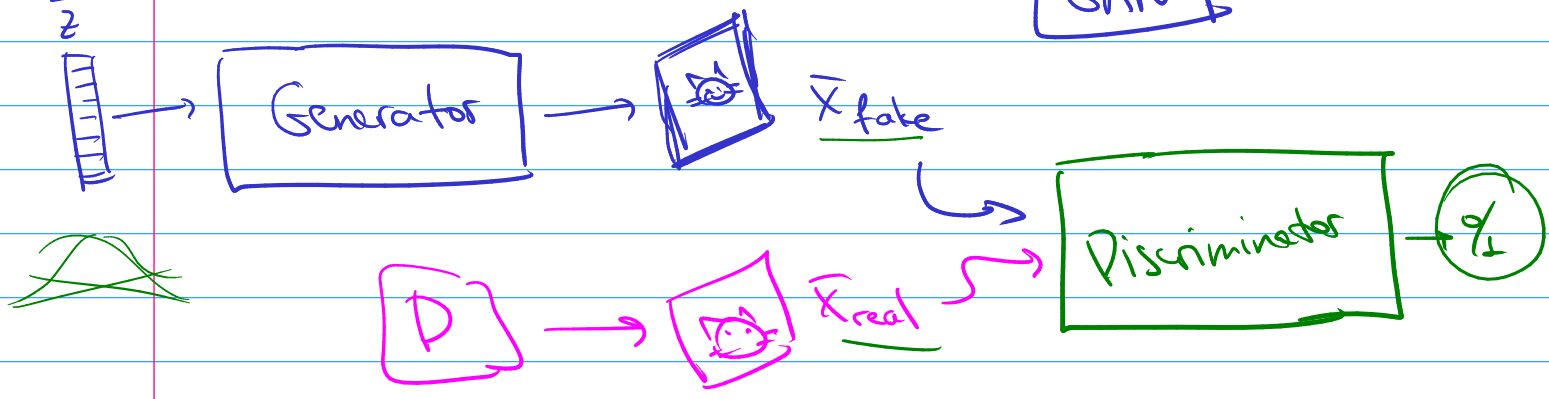
Implicit density



denoising Autoencoder



GAN



$$L_D(\bar{\theta}_D) = \mathbb{E}_{\bar{x}_{real}}[\log D(\bar{x})] + \mathbb{E}_{\bar{x}_{fake}}[\log(1 - D(\bar{x}))] \xrightarrow[\bar{\theta}_D]{} \max$$

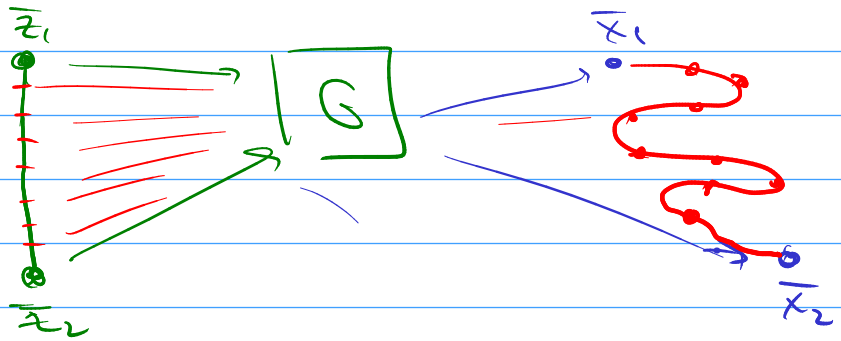
$$L_G(\bar{\theta}_G) = \mathbb{E}_{\bar{z}}\left[\log\left(1 - D(G(\bar{z}))\right)\right] \xrightarrow[\bar{\theta}_G]{} \min$$

$$\min_{\bar{\theta}_G} \max_{\bar{\theta}_D} L(\bar{\theta}_D, \bar{\theta}_G)$$

fix G $\Rightarrow$ $D^*_G(\bar{x}) = \dfrac{p_{data}(\bar{x})}{p_{data}(\bar{x}) + p_g(\bar{x})}$ $\Rightarrow$

$\Rightarrow$ G^* : $p_g(\bar{x}) = p_{data}(\bar{x})$

$$L_G = JSD(p_g \| p_{data}) = KL\left(p_{data} \| \frac{p_{data}+p_g}{2}\right) + KL\left(p_g \| \frac{p_{data}+p_g}{2}\right)$$



LSGAN — least squares

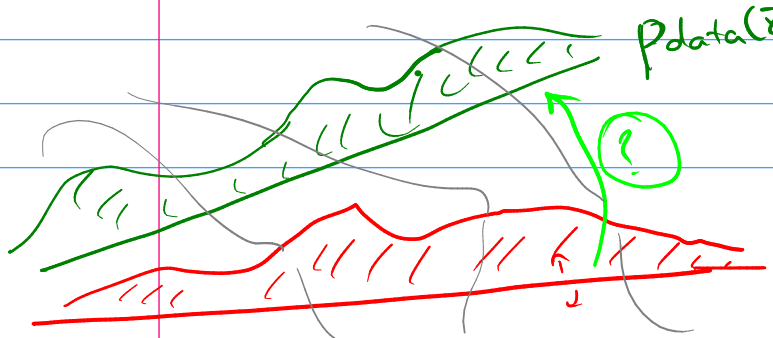
$$L(D) = \mathbb{E}_{\bar{x}\sim p_g}\left[(D(\bar{x}) - a)^2\right] + \mathbb{E}_{\bar{x}\sim p_{data}}\left[(D(\bar{x}) - b)^2\right]$$

$$L(G) = \mathbb{E}_{\bar{z}}\left[(D(G(\bar{z})) - c)^2\right]$$

$a = 0$, $b = c = 1$

$c = \frac{a+b}{2}$

WGAN - Wasserstein GAN

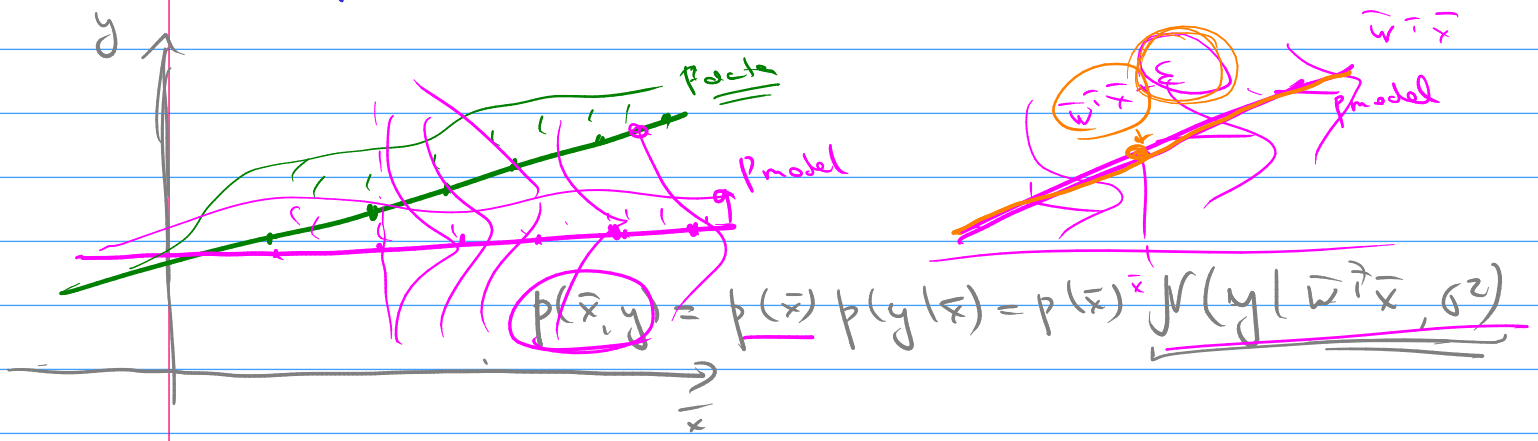


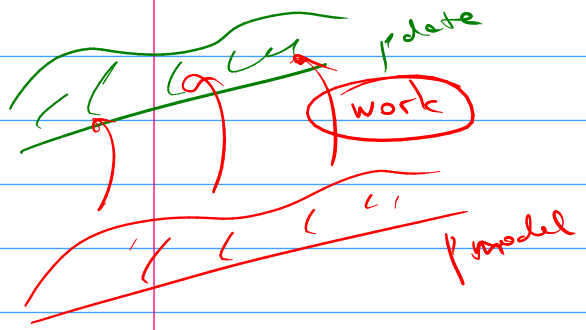
$$KL(p\|q) = \int p(\bar{x}) \ln \frac{p(\bar{x})}{q(\bar{x})} d\bar{x}$$

$KL(p_{model} \| p_{data}) = \infty$

$KL(p_{data} \| p_{model}) = \infty$

$JSD(p_{data} \| p_{model}) = KL\left(p_{data} \| \frac{p_{data} + p_{model}}{2}\right) + \_ = \text{const}$

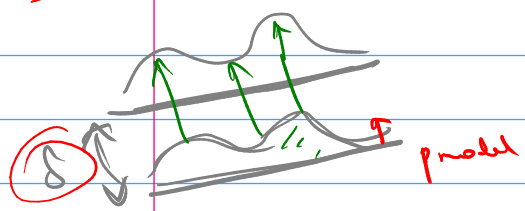


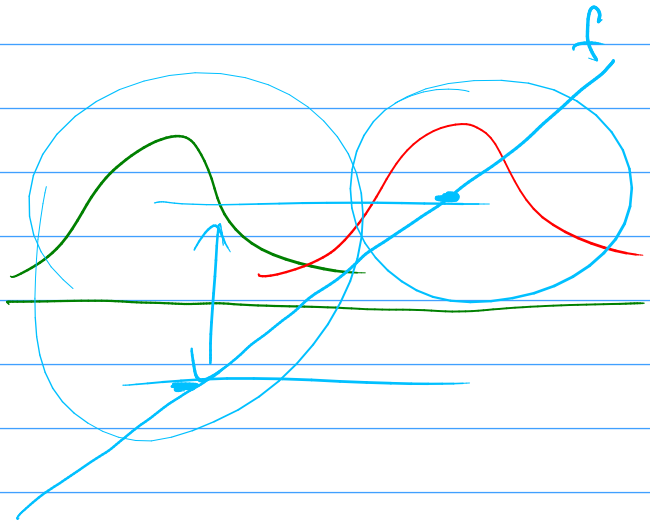


$\bar{x}, \bar{z} \rightarrow \rightarrow y \sim \mathcal{N}(y | \bar{w}^T \bar{x}, \sigma^2)$

Earth Mover Distance

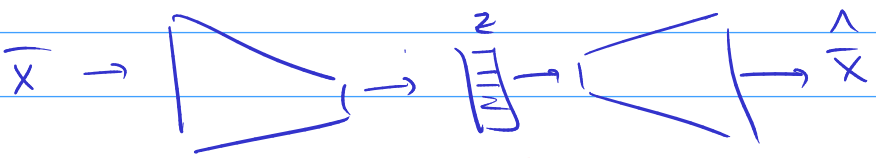
Wasserstein distance

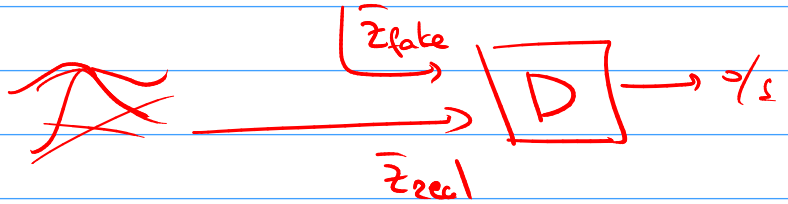




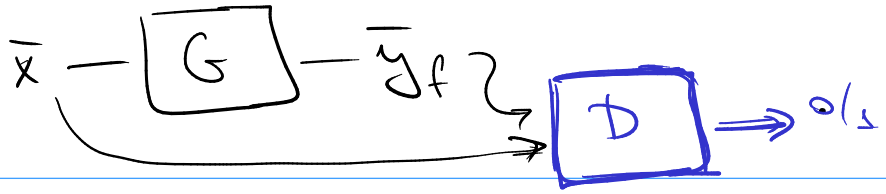
AAE

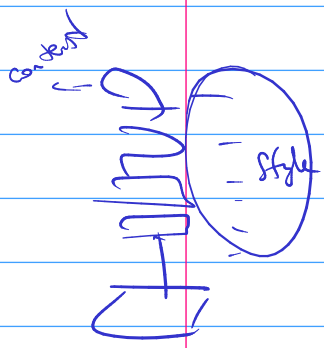
adversarial autoencoder





pix2pix





$(\bar{x}_{real}, \bar{y}_{real})$

paired data

$$\gamma \left(\frac{\bar{x} - \mu(\bar{x})}{\sigma(\bar{x})} \right) + \beta$$

CycleGAN