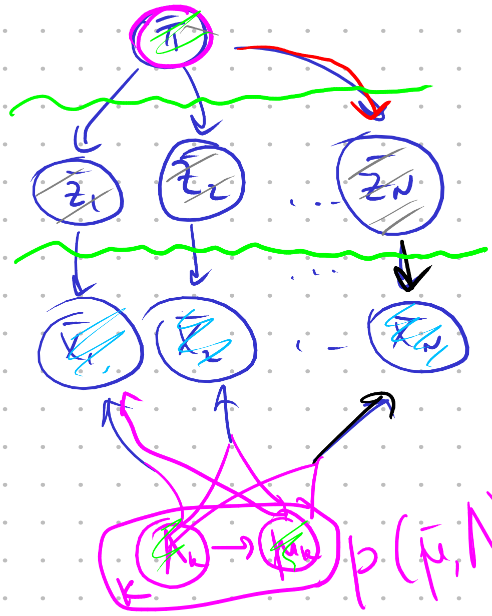



$$p(Z|\bar\pi) = \prod_n p(\bar z_n|\bar\pi) = \prod_n \prod_k \pi_k^{z_{nk}}$$

$$p(X|Z,\bar\mu,\Lambda) = \prod_n p(\bar x_n|\bar z_n,\bar\mu,\Lambda) =$$

$$= \prod_n \prod_k N(\bar x_n|\bar\mu_k,\Lambda_k)^{z_{nk}}$$

$$p(\bar\pi) = Dir(\bar\pi|\alpha_0) = \dots \prod_{k=1}^{K} \pi_k^{\alpha_0 - 1}$$

$$p(\bar\mu,\Lambda) = \prod_k p(\bar\mu_k,\Lambda_k) = \prod_k p(\Lambda_k)\, p(\bar\mu_k|\Lambda_k) =$$

$$= \prod_k W(\Lambda_k|W_0,\nu_0)\, N(\bar\mu_k|\underset{=0}{\bar m_0}, \beta_0\Lambda_k)$$

$$p(X,Z,\pi,\mu,\Lambda) = p(\bar\pi)\cdot\left(\prod_k p(\Lambda_k)\, p(\bar\mu|\Lambda_k)\right)\cdot p(Z|\pi)\, p(X|Z,\pi,\mu,\Lambda)$$

$$q(Z,\bar\pi,\mu,\Lambda) = q(Z)\, q(\bar\pi,\mu,\Lambda) \simeq p(Z,\pi,\mu,\Lambda|X)$$

$$\ln q^*(Z) = \mathbb{E}_{\pi,\mu,\Lambda}\left[\ln p(X,Z,\pi,\mu,\Lambda)\right] + const$$

$$\ln q^*(\pi,\mu,\Lambda) = \mathbb{E}_Z\left[\ln p(X,Z,\pi,\mu,\Lambda)\right] + const$$

$$\ln q^*(Z) = \mathbb{E}_\pi\left[\ln p(Z|\pi)\right] + \mathbb{E}_{\mu,\Lambda}\left[\ln p(X|Z,\pi,\mu,\Lambda)\right] + const$$

$$= \mathbb{E}_\pi\left[\sum_n \sum_k z_{nk} \ln \pi_k\right] + \mathbb{E}_{\mu,\Lambda}\left[\sum_n \sum_k z_{nk}\left(\frac{1}{2}\ln\det\Lambda_k - \frac{d}{2}\ln 2\pi - \frac{1}{2}(\bar x_n - \bar\mu_k)^T \Lambda_k (\bar x_n - \bar\mu_k)\right)\right] =$$

$$= \sum_n \sum_k z_{nk} \underbrace{\mathbb{E}_{\pi,\mu,\Lambda}[\dots]}_{\ln \rho_{nk}} + const$$

$$\ln \rho_{nk} = E_\pi[\ln \pi_k] + \frac{1}{2}\mathbb{E}_\Lambda[\ln\det\Lambda_k] - \frac{1}{2}\mathbb{E}_{\mu,\Lambda}\left[(\bar x_n - \bar\mu_k)^T \Lambda_k (\bar x_n - \bar\mu_k)\right]$$

$$\ln q^*(Z) = \sum_n \sum_k z_{nk} \ln \rho_{nk} + \text{const}$$

$$\Rightarrow \quad \mathbb{E}[z_{nk}] = \frac{\rho_{nk}}{\sum_l \rho_{nl}} = r_{nk}$$

$$\ln q^*(\pi,\mu,\Lambda) = \mathbb{E}_Z[\ln p(X,Z,\pi,\mu,\Lambda)] + \text{const} =$$

$$= \ln p(\pi) + \underline{\sum_k} \ln p(\mu_k,\Lambda_k) + \sum_n \underline{\sum_k} (\ln \pi_k) \cdot \mathbb{E}[z_{nk}] +$$

$$+ \sum_n \underline{\sum_k} (\mathbb{E} z_{nk}) \ln p(x_n \mid \mu_k, \Lambda_k) + \text{const}$$

$$q^*(\pi,\mu,\Lambda) = q^*(\pi) \cdot \prod_{k=1}^{K} q^*(\mu_k,\Lambda_k)$$

$$\ln q^*(\pi) = \overbrace{\ln p(\pi)}^{\text{const} + \sum (\alpha_0 - 1) \ln \pi_k} + \sum_n \sum_k \mathbb{E}[z_{nk}] (\ln \pi_k) + \text{const} =$$

$$= \sum_k \left(\alpha_0 - 1 + \sum_n \mathbb{E}[z_{nk}]\right) \ln \pi_k + \text{const}$$

$$q^*(\pi) = \text{Dir}(\pi \mid \alpha), \text{ ige } \alpha_k = \alpha_0 + \sum_n r_{nk}$$

$$q^*(\mu_k,\Lambda_k) = q^*(\Lambda_k)\, q^*(\mu_k \mid \Lambda_k) = \mathcal{W}(\Lambda_k \mid W_k, \nu_k) \cdot \mathcal{N}(\mu_k \mid m_k, \beta_k \Lambda_k)$$

$\alpha_k = \alpha_0 + \sum_n r_{nk}$
$W_k = \ldots$
$\nu_k = \ldots$
$m_k = \ldots$
$\beta_k = \ldots$

Variational EM

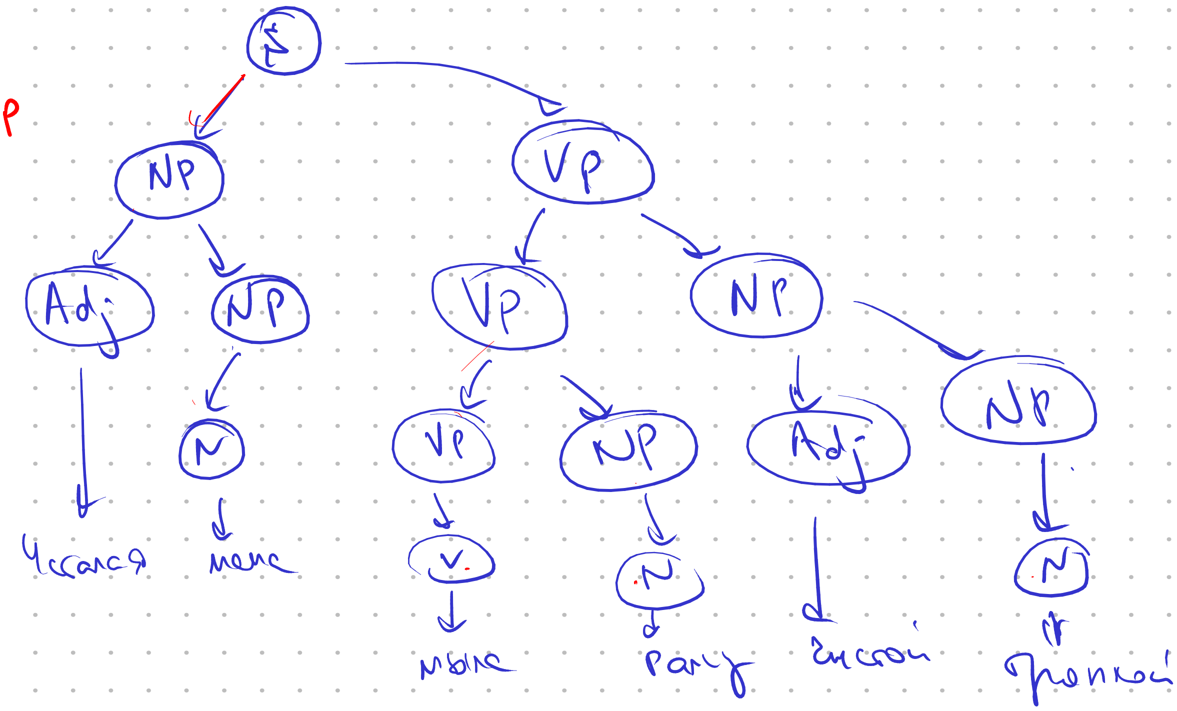
$q(\pi,\mu,\Lambda) \longrightarrow r_{nk}$

$q(\pi,\mu,\Lambda) \longrightarrow r_{nk}$

$\ldots$

Усталая мама мыла раму чистой тряпкой

S → NP VP
NP → Adj NP
NP → N
VP → VP NP
VP → V




Adj N V N Adj N

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| Pron | N | V | N | Adj | Adv |
|---|---|---|---|---|---|
| Наша | мама | мыла | раму | ранним | утром |

вымытый

lemmatization

stemming

Person

| Великий | поэт | А. | С. | Пушкин | (р. | 1799 | г.) | умер | в | Санкт-Петербурге. |
|---|---|---|---|---|---|---|---|---|---|---|
| O | O | B-Per | Per | Per | O | B-Date | Date | O | O | B-Loc |