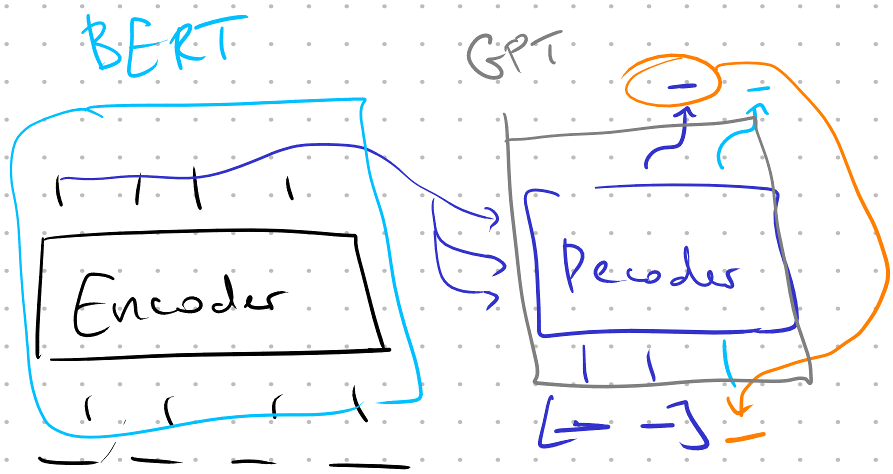

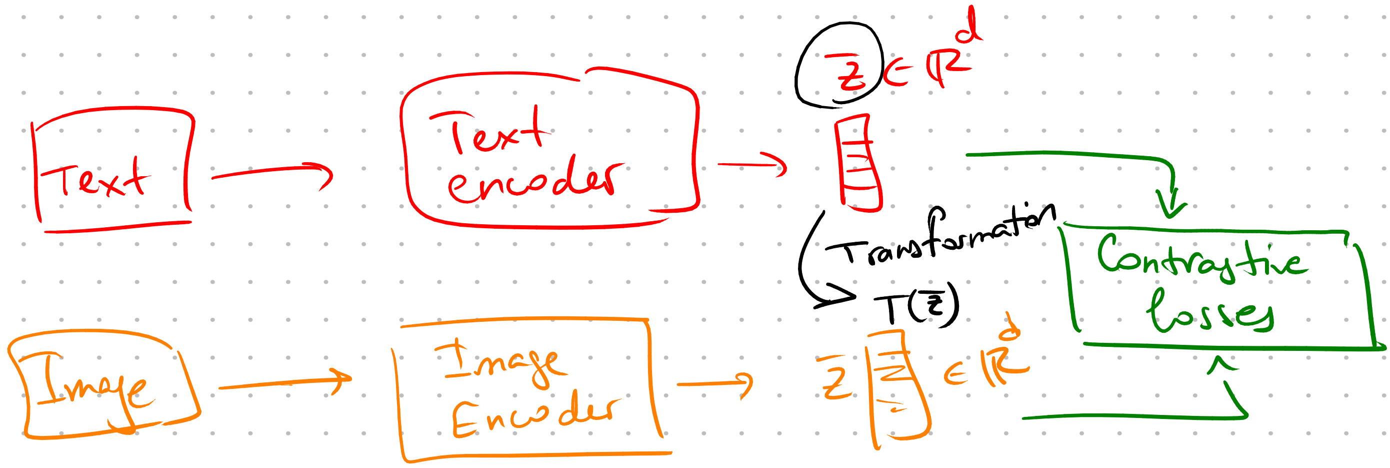

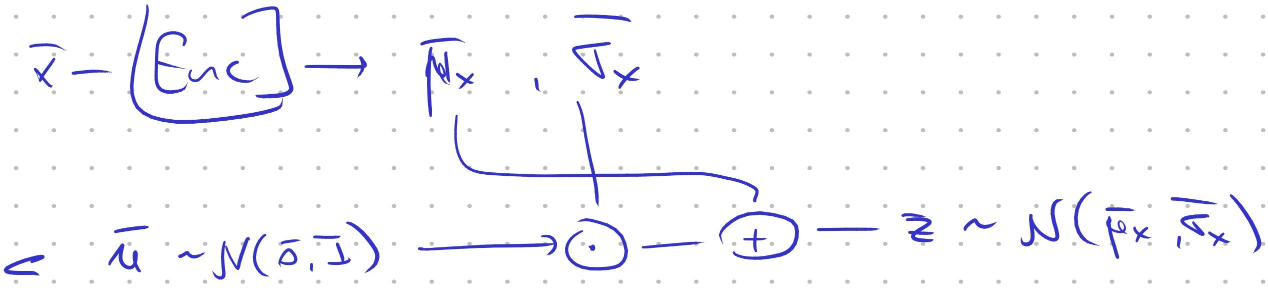

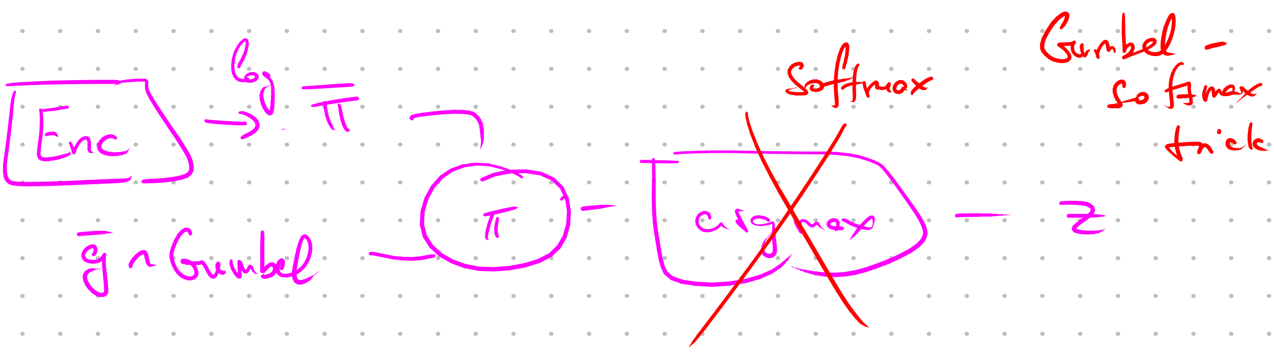

Gumbel - Max trick

Thm. $z \sim \text{Mult}(\bar{\pi}) \iff z = \arg\max_i \left(g_i + \log \pi_i\right)$,

rge $g_i \sim \text{Gumbel}$

$$p(g_i) = e^{-(g_i + e^{-g_i})}, \quad F(g_i) = e^{-e^{-g_i}}$$

1 bo:

$$p(z=k) = p(\forall j \;\; g_k + \log\pi_k \geq g_j + \log\pi_j) =$$

$$= \int_{-\infty}^{\infty} p(g_k) \cdot \prod_{j\neq k} \underbrace{p(g_j + \log\pi_j \leq g_k + \log\pi_k)}_{= F(g_k + \log\pi_k - \log\pi_j)} \cdot dg_k =$$

$$= \int_{-\infty}^{\infty} \left(\prod_{j\neq k} e^{-e^{-g_k - \log\pi_k + \log\pi_j}} \right) \cdot e^{-g_k - e^{-g_k \pm \log\pi_k}} \; \pm \log\pi_k \; dg_k =$$

$$= \int_{-\infty}^{\infty} e^{-\sum_{j\neq k}\pi_j e^{-g_k - \log\pi_k}} \cdot \pi_k \cdot e^{-g_k - \log\pi_k - \pi_k e^{-g_k - \log\pi_k}} \, dg_k$$

$$= \pi_k \underbrace{\int_{-\infty}^{\infty} e^{-g_k - \log\pi_k - e^{-g_k - \log\pi_k}\big(\overbrace{\pi_k + \sum_{j\neq k}\pi_j}^{=1}\big)} \, dg_k}_{=1} = \pi_k$$