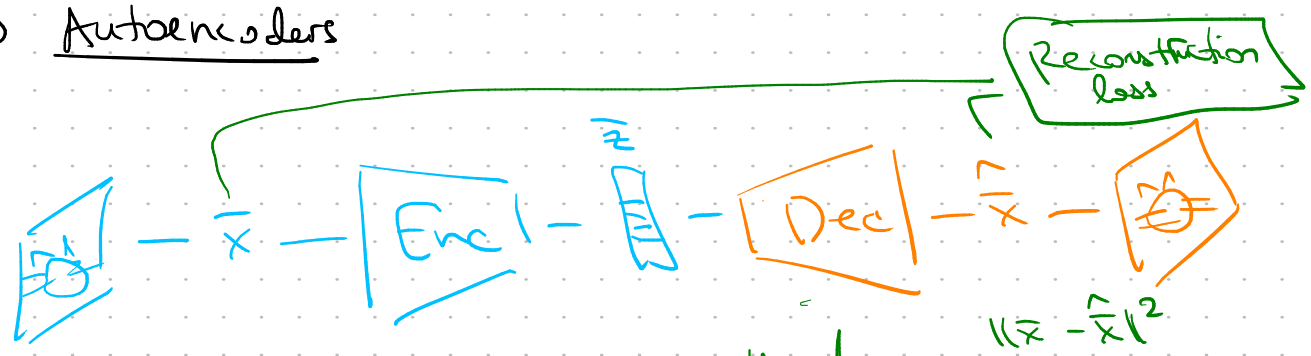
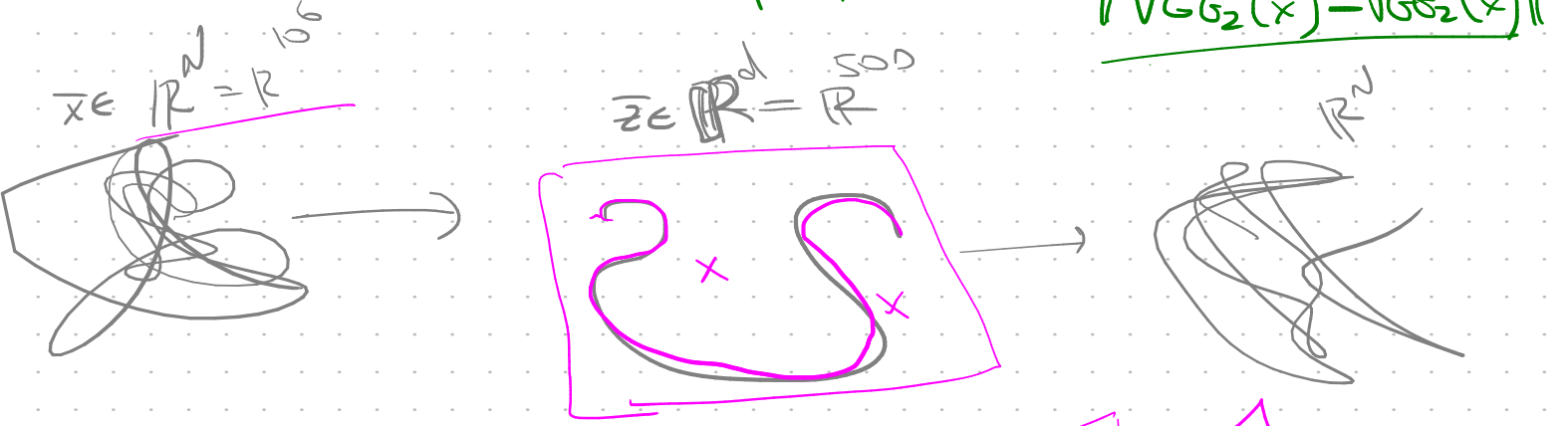
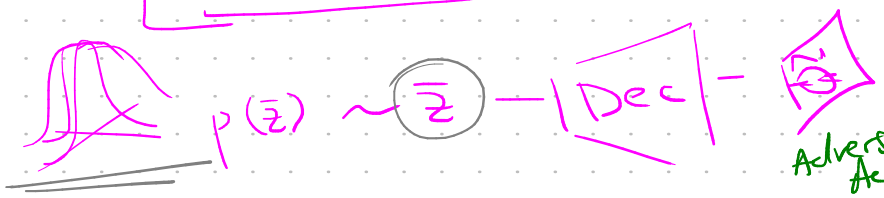
① Autoencoders

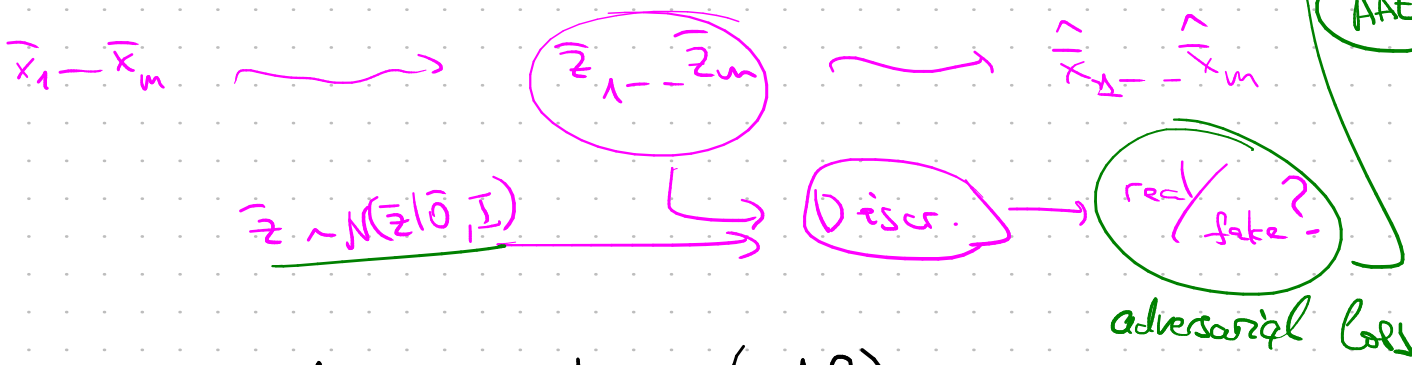






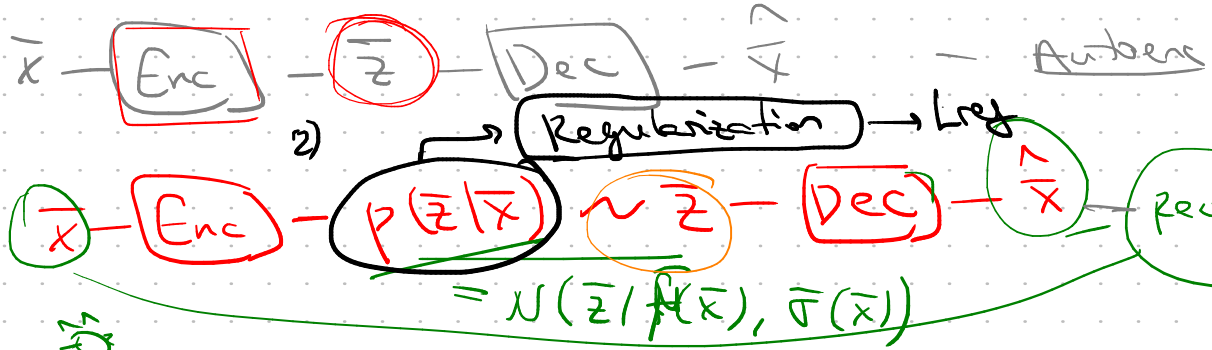
Adversarial Autoencoders

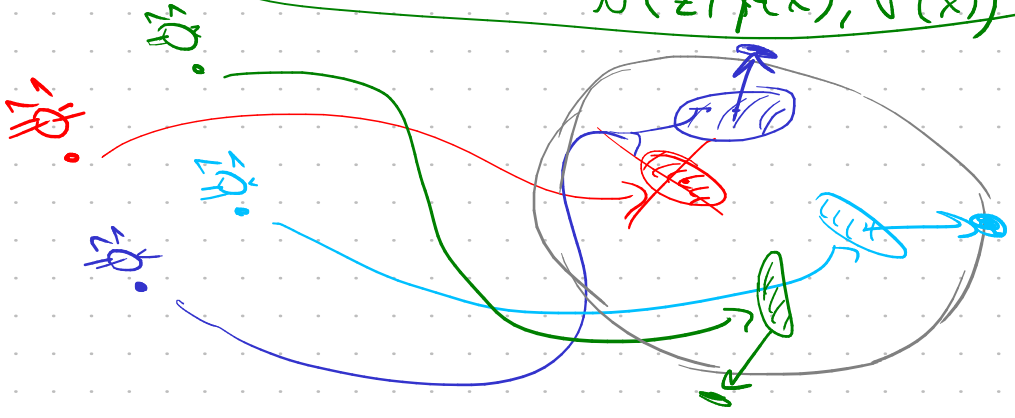
AAE



adversarial loss

② Variational autoencoder (VAE)





Reg:

$p(\bar{z}|\bar{x}) \approx \mathcal{N}(\bar{z}|\bar{0}, \bar{I})$

$\mathcal{N}(z|0,I)$ Decoder

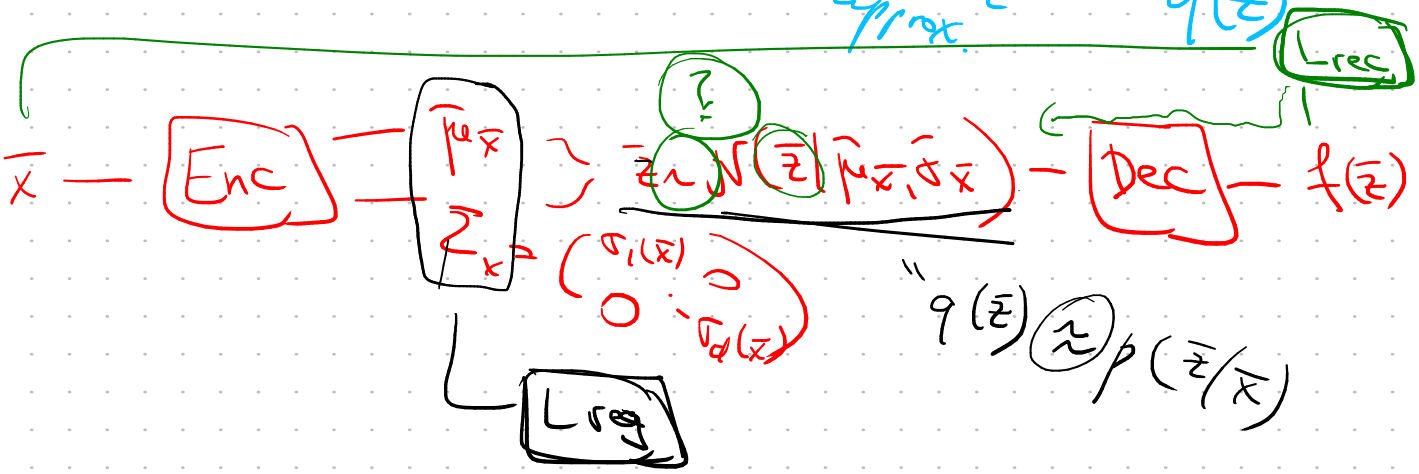
$p(z) \cdot p(x|z) = p(x,z) = p(x) p(z|x)$

$\mathbb{R}^N$ Encoder

$\mathcal{N}(x|\text{Dec}(z), c\cdot I)$

$\mathcal{N}(z|\mu_x, \sigma_x)$

Variational approx. $q(z)$



(3) Var. appr.

$\log p(x,z) = \log p(x) p(z|x)$

$\log p(x) = \log p(x,z) - \log p(z|x) \quad | \; \mathbb{E}_{q(z)}$

$\log p(x) = \mathbb{E}_q[\ldots - \ldots] \pm \mathbb{E}_q[\log q(z)]$

$\log p(x) = \mathbb{E}_{q(z)}\left[\log \frac{p(x,z)}{q(z)}\right] + \mathbb{E}_{q(z)}\left[\log \frac{q(z)}{p(z|x)}\right]$

$\text{Const} = L(q) + KL(q(z) \| p(z|x))$

$L(q)$: ELBO / variational lower bound

$q \to \max$

$q \to \min$

$= p(z) p(x|z)$

$L(q) = \int q(z) \log \frac{p(x,z)}{q(z)} dz =$

$= \int q(z) \log p(x|z) dz - \int q(z) \log \frac{q(z)}{p(z)} dz$

$$\mathbb{E}_{q(z)}\left[\log \mathcal{N}(x \mid f(z), cI)\right] \qquad KL(q(z) \,\|\, p(z)) \qquad p(z) = \mathcal{N}(z \mid 0, I)$$

$$\mathbb{E}_{q(z)}\left[\text{Const} - \frac{1}{2c^2}\cdot \|x - f(z)\|^2\right]$$

Regularizer

$$\approx \text{Const} - \text{Const}\cdot \sum_n \|x_n - \text{Dec}(\text{Enc}(x_n))\|^2$$

Reconstruction loss

$$\max \uparrow \quad L(q) \approx \text{Const} - \left(c' \cdot \sum_n \|x_n - \text{Dec}(\text{Enc}(x_n))\|^2 + KL(q \,\|\, p(z)) \right)$$

$$c' L_{rec} + L_{reg} \quad \downarrow \min$$

$$\left(\begin{matrix}\mu_1 \\ \sigma_1^2\end{matrix}\right) \quad \left(\begin{matrix}\mu_2 \\ \sigma_2^2\end{matrix}\right)$$

$$q(z) = \mathcal{N}(z \mid \mu_x, \sigma_x)$$

$$KL\left(\prod_i q_i(z_i) \,\|\, \prod_i p_i(z_i)\right) = \sum_{i=1}^{d} KL(q_i(z_i) \,\|\, p_i(z_i))$$

$$KL(\mathcal{N}(z_i \mid \mu_{x_i}, \sigma_{x_i}^2) \,\|\, \mathcal{N}(z_i \mid 0, 1)) =$$

$$= \mathbb{E}_{N_x}\left[\log \frac{N_x}{N}\right] = \mathbb{E}_{N_x}\left[-\frac{1}{2}\log \sigma_{x_i}^2 - \frac{1}{2\sigma_{x_i}^2}(z_i - \mu_{x_i})^2 + \frac{1}{2} z_i^2 \right] =$$

$$= \mathbb{E}_{N_x}\left[z_i^2 \left(\frac{1}{2} - \frac{1}{2\sigma_{x_i}^2}\right) + z_i \frac{\mu_{x_i}}{\sigma_{x_i}^2} \right] - \frac{1}{2}\log \sigma_{x_i}^2 - \frac{\mu_{x_i}^2}{2\sigma_{x_i}^2} =$$

($\mathbb{E}[z_i^2] = \mu_{x_i}^2 + \sigma_{x_i}^2$, $\mathbb{E}[z_i] = \mu_{x_i}$)

$$= \frac{1}{2}\mu_{x_i}^2 - \frac{\mu_{x_i}^2}{2\sigma_{x_i}^2} + \frac{1}{2}\sigma_{x_i}^2 = \frac{1}{2} + \frac{\mu_{x_i}^2}{\sigma_{x_i}^2} - \frac{1}{2}\log \sigma_{x_i}^2 - \frac{\mu_{x_i}^2}{2\sigma_{x_i}^2}$$

$$KL(q_i \| p_i) = \frac{1}{2}\left(\mu_{x_i}^2 + \sigma_{x_i}^2 - \log \sigma_{x_i}^2 - 1\right)$$

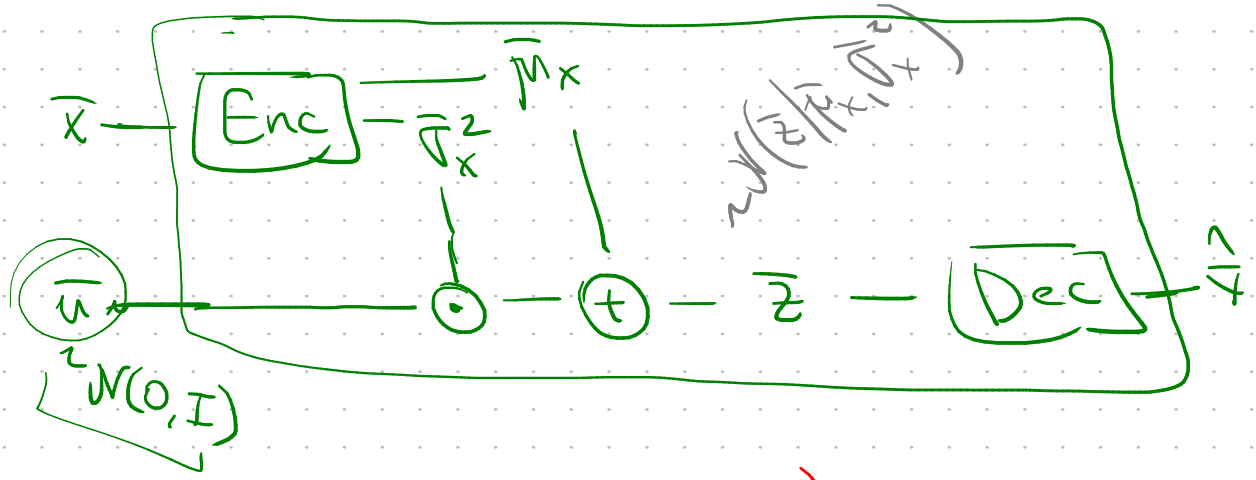
$$L_{reg} = KL(q\|p) = \sum_{i=1}^{d} \frac{1}{2}\left(\mu_{x_i}^2 + \sigma_{x_i}^2 - \log \sigma_{x_i}^2 - 1\right)$$

(3) Reparametrization trick

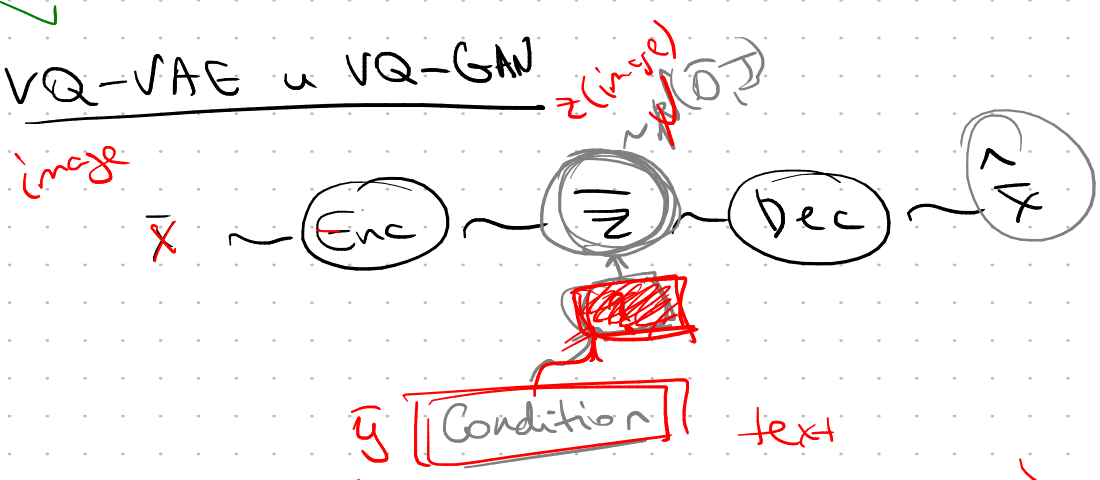
$\bar{x}$ — Enc — $\bar{\mu}_x$, $\bar{\sigma}_x^2$ ⇝ $\bar{z} \sim N(\bar{z} | \bar{\mu}_x, \bar{\sigma}_x^2)$ — Dec — $\hat{\bar{x}}$

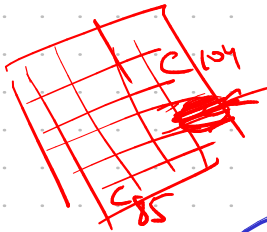
$z_i \sim N(z_i | \mu_{x_i}, \sigma_{x_i}^2) \Leftrightarrow z_i = u_i \cdot \sigma_{x_i} + \mu_{x_i}$, где

$u_i \sim N(u_i | 0, 1)$



(4) VQ-VAE и VQ-GAN



$\bar{z}$: 

$D = \{(\bar{y}_i, \bar{x}_i)\}$

$\bar{y}_i$ — [ ] ⇝ $Enc(\bar{x}_i)$ $c_1 c_{85} c_{104}$

$\bar{z}$: 32×32 32^2 2048

$|V| = 2048$