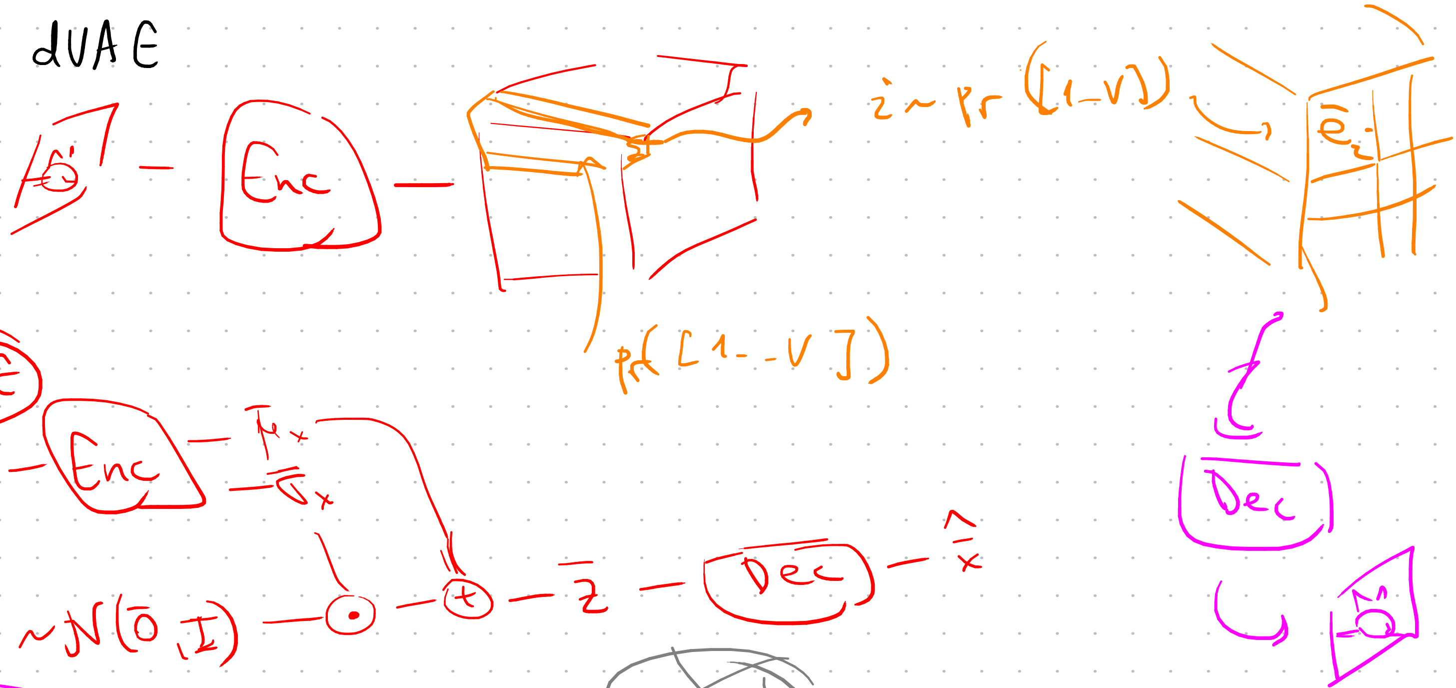
① dVAE



VAE

$\bar{x}$ — Enc — $\bar{\mu}_x$, $\bar{\sigma}_x$

$\bar{u} \sim N(\bar{0}, I)$ — $\odot$ — $\oplus$ — $\bar{z}$ — Dec — $\hat{x}$

dVAE

~~sample~~

$\bar{x}$ — Enc — $\bar{\pi} = (\pi_1 \dots \pi_K)$ — $i \sim Pr_{\bar{\pi}}$ — $\bar{e}_i$ — Dec — $\hat{x}$

$\bar{\pi}' = (\_ \quad \_)$ — $j \sim Pr_{\bar{\pi}'}$ — $\bar{e}_j$

Gumbel-Max trick:

$z \sim Mult(\bar{\pi})$

$\pi = (\pi_1 \dots \pi_K)$

$z = \underbrace{\arg\max_i (g_i + \log \pi_i)}$, где $g_i \sim Gumbel$

$p(g_i) = e^{-(g_i + e^{-g_i})}$, $F(g_i) = e^{-e^{-g_i}}$

$\bar{x}$ — Enc — $\bar{\pi}$ — log — $\oplus$ — argmax — i — $\bar{e}_i$ — Dec

$g \sim Gumbel$

Proof

$p(z=k) = p(g_k + \log \pi_k \geq g_j + \log \pi_j \ \forall j) =$

$= \int_{-\infty}^{\infty} \underline{p(\forall j \neq k \ g_j + \log \pi_j \leq g_k + \log \pi_k)} \, p(g_k) \, dg_k =$

$\prod_{j \neq k} p(g_j \leq g_k + \log \pi_k - \log \pi_j \mid g_k) = \prod_{j \neq k} F(g_k + \log \pi_k - \log \pi_j)$

$$= \int_{-\infty}^{\infty} \left(\prod_{j \neq k} e^{-e^{-g_k - \log \pi_k + \log \pi_j}} \right) \cdot e^{-(g_k + e^{-g_k})} \, dg_k =$$

$$= \int_{-\infty}^{\infty} \prod_{j \neq k} e^{-\pi_j \cdot e^{-g_k - \log \pi_k}} \cdot e^{-(g_k + e^{-g_k})} \, dg_k =$$

$$= \int_{-\infty}^{\infty} e^{-e^{-g_k - \log \pi_k} \left(\sum_{j \neq k} \pi_j \; +\pi_k - \pi_k \right)} \cdot e^{-g_k - e^{-g_k}} \, dg_k$$

$$= \int_{-\infty}^{\infty} e^{-e^{-g_k - \log \pi_k} + \pi_k \cdot e^{-g_k - \log \pi_k} - g_k - e^{-g_k}} \, dg_k$$

$$-e^{-(g_k + \log \pi_k)} - (g_k + \log \pi_k) + \cancel{g_k} + \log \pi_k + \cancel{\pi_k \cdot e^{-g_k - \log \pi_k}} - \cancel{g_k} - \cancel{e^{-g_k}}$$

$$= e^{\log \pi_k} \int_{-\infty}^{\infty} e^{-(g_k + \log \pi_k) - e^{-(g_k + \log \pi_k)}} \, dg_k = \pi_k$$

Gumbel-Softmax:

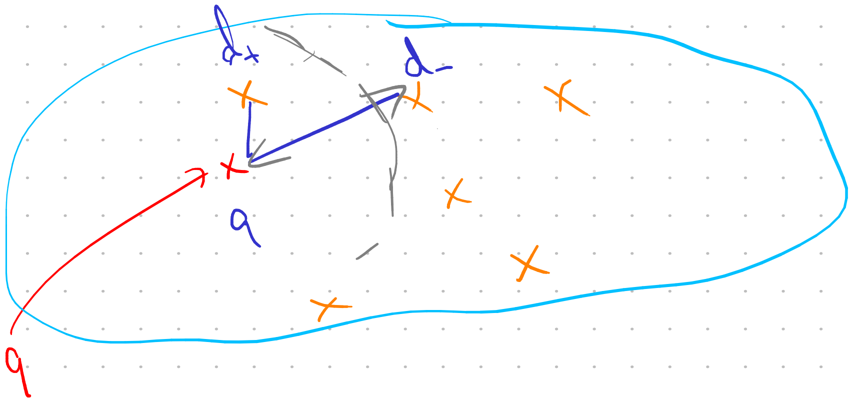
$$\bar{x} \to \boxed{\text{Enc}} \to \bar{\pi} \to \boxed{\log} \to \oplus \to \boxed{\text{softmax}} \to \bar{\alpha} \qquad \bar{z} = \sum \alpha_i e_i$$

$$\bar{g} \sim \text{Gumbel} \to \oplus$$

$$\alpha_i = \text{softmax}\left(\frac{1}{\tau} \left(\bar{g} + \log \bar{\pi} \right) \right)_i = \frac{e^{\frac{1}{\tau}(g_i + \log \pi_i)}}{\sum_j e^{\frac{1}{\tau}(g_j + \log \pi_j)}}$$

temperature

Contrastive learning



$$L(\bar{q}, \bar{d}_+, \bar{d}_-) = \bar{q}^T \bar{d}_+ - \bar{q}^T \bar{d}_-$$

triplet loss