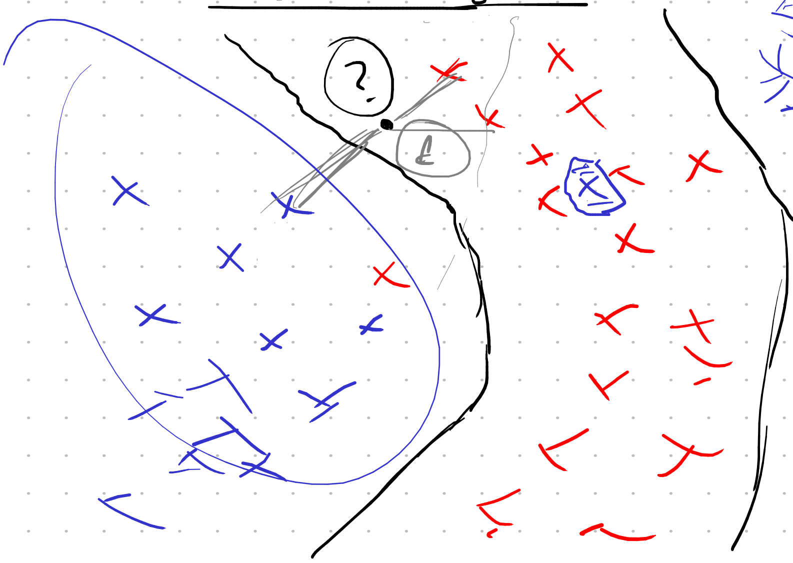
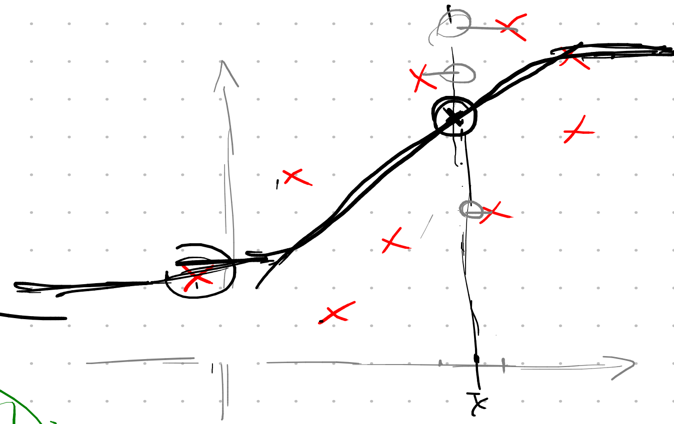
Nearest neighbors:



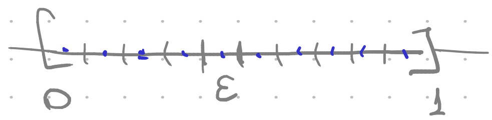


non-parametric

Curse of dimensionality

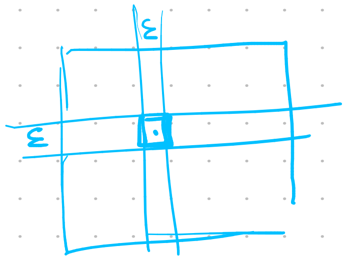
① Численное интегрирование

$d=1$



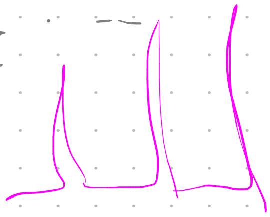
$$\int_0^1 f(\bar{x})dx \approx \sum_1^{1/\varepsilon} \dots$$

$d=2$



$1/\varepsilon^2$

d $\quad 1/\varepsilon^d$



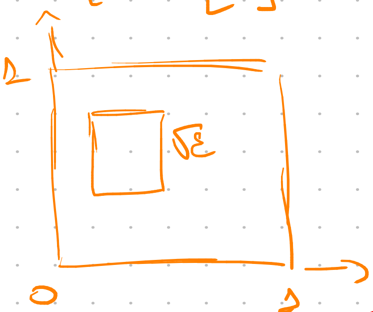
$$p(\bar{\theta}|D) \propto p(\bar{\theta})\,p(D|\bar{\theta})$$

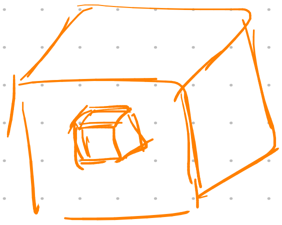
$$p(\bar{x}|D) = \int p(\bar{x}|\bar{\theta})\,p(\bar{\theta}|D)\,d\bar{\theta} =$$

$$= \mathbb{E}_{p(\bar{\theta}|D)}\left[p(\bar{x}|\bar{\theta})\right]$$

MCMC

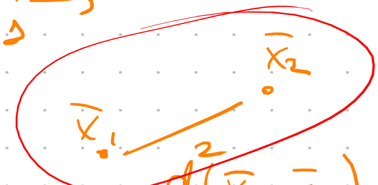
②



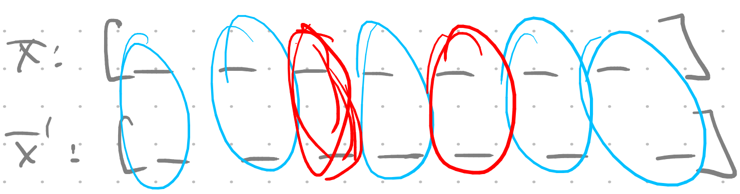


$$\sqrt[d]{\varepsilon} = \varepsilon^{1/d}$$

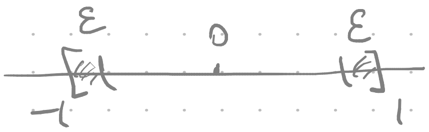
N точек



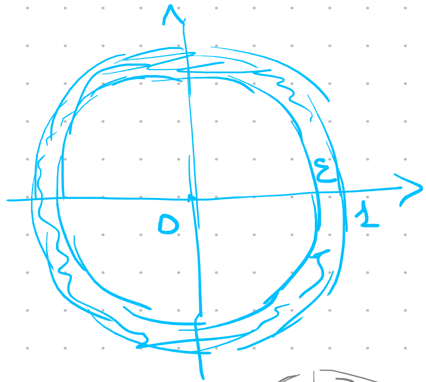
$$d^2(\bar{x}_1, \bar{x}_2) = \sum_i^{d} (x_{1i} - x_{2i})^2$$



3 Эффект кожуры апельсина

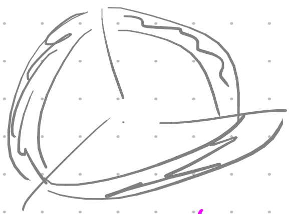


ε



πr^2

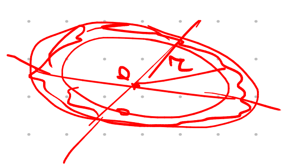
$$\frac{\pi\cdot 1^2 - \pi(1-\varepsilon)^2}{\pi\cdot 1^2} = \boxed{1-(1-\varepsilon)^2} = 2\varepsilon-\varepsilon^2$$



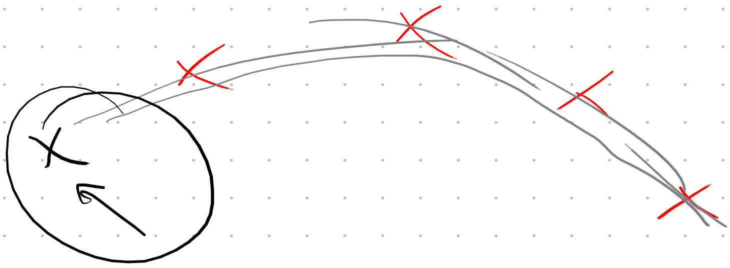
$$\frac{\frac{4}{3}\pi\cdot 1^3 - \frac{4}{3}\pi\cdot(1-\varepsilon)^3}{4/3\,\pi\cdot 1} = 1-(1-\varepsilon)^3$$

$$1-(1-\varepsilon)^d \xrightarrow[d\to\infty]{} 1$$

$$\bar x \sim N(\bar x|\bar 0, I)$$



$$r^2 = \sum x_i^2 \sim N(x|0,1)$$



Statistical decision theory

$\bar x \in \mathbb{R}^d$ $\quad y\in\mathbb{R}$ $\quad D=\{(\bar x_n, y_n)\}_{n=1}^N$ $\quad p(y|\bar x)$

2

$$p(\bar x, y) = p(\bar x)\,p(y|\bar x)$$

$$L(y, f(\bar x)) = (y-f(\bar x))^2$$

exp-pred-error

$$EPE[f] = \mathbb{E}_{p(\bar x, y)}[L(y, f(\bar x))] =$$

$$= \iint (y-f(x))^2 p(y|\bar x)\,p(\bar x)\,dy\,d\bar x =$$

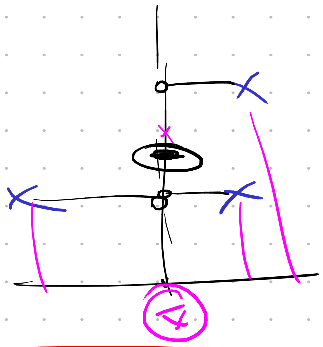
$$= \int \left[\int (y-f(\bar x))^2 p(y|\bar x)\,dy\right] p(\bar x)\,d\bar x \xrightarrow[f]{} \min$$

$$p(\bar\theta|D) = \frac{p(\bar\theta)\,p(D|\bar\theta)}{p(D)}$$

$$p(\bar w|\bar y, X) = \frac{p(\bar w)\,p(\bar y|\bar w, X)}{p(\bar y|X)}$$

$f(\bar{x}) \to \min$

$$\boxed{\hat{f}(\bar{x}) = \mathbb{E}_{p(y|\bar{x})}[y]}$$ — regression function



$$\hat{f}(\bar{x}) = \mathbb{E}_{p(y|\bar{x})}[y] \approx$$

$$\approx \frac{1}{N}\sum_{n=1}^{N} y_n \quad \text{где } y_n \text{ взяты из } p(y|\bar{x})$$

$$\approx \frac{1}{N}\sum_{n=1}^{N} y_n, \quad \text{где } y_n \in \underline{N\text{-}NN(\bar{x})}$$

$\bar{x} \in \mathbb{R}^d \quad y \in \{C_1 \dots C_K\}$

$$L(y, f(\bar{x})) = \begin{cases} 0, & y = f(\bar{x}) \\ 1 & y \neq f(\bar{x}) \end{cases}$$

$$EPE[f] = \iint L(y, f(\bar{x}))\, p(\bar{x}, y)\, dy\, d\bar{x}$$

$$= \int \underbrace{\left[\sum_{k=1}^{K} L(C_k, f(\bar{x}))\, p(C_k|\bar{x})\right]}\, p(\bar{x})\, d\bar{x}$$

$f(\bar{x}) \to \min$

$$\boxed{\hat{f}(\bar{x}) = \arg\max_k p(C_k|\bar{x})}$$ optimal Bayes classifier

$$\boxed{\hat{f}(\bar{x}) = \arg\max_s \sum_{k=1}^{K} L(C_k, s)\, p(C_k|\bar{x})}$$

Disease \ Test	0	1
0	0	1
1	1000	0

$$\left((y - \hat{f}) + (\hat{f} - f)\right)^2$$

$$EPE[f] = \iint (y - f(\bar{x}))^2 \, p(\bar{x}, y) \, d\bar{x} \, dy \qquad \pm \hat{f}(\bar{x})$$

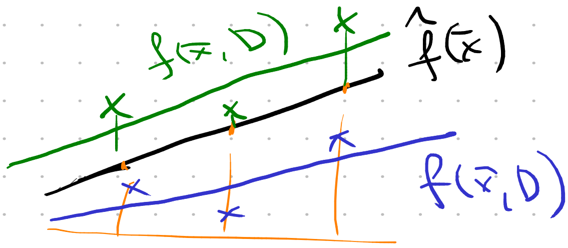
$$= \iint \left((y - \hat{f}(\bar{x}))^2 - 2(y - \hat{f}(\bar{x}))(\hat{f}(\bar{x}) - f(\bar{x})) + (\hat{f}(\bar{x}) - f(\bar{x}))^2 \right) p(\bar{x}, y) \, d\bar{x} \, dy$$

$$= E_p[(y - \hat{f})^2] - 2 \cancel{E_p[(y - \hat{f})(\hat{f} - f)]} + E_p[(\hat{f} - f)^2]$$

$$\iint (y - \hat{f}(\bar{x}))(\hat{f}(\bar{x}) - f(\bar{x})) \, p(y|\bar{x}) \, p(\bar{x}) \, dy \, d\bar{x}$$

$$\cancel{\int (y - E_y) p \, dy} \qquad = \int \left[\int (y - \hat{f}(\bar{x})) p(y|\bar{x}) \, dy \right] \ldots \, d\bar{x}$$

$$EPE[f] = \underbrace{E[(y - \hat{f}(\bar{x}))^2]}_{\text{Noise}} + E[(\hat{f}(\bar{x}) - f(\bar{x}))^2]$$



$$f(\bar{x}; D), \quad D \sim p(\bar{x}, y)$$

$$E_D[f(\bar{x})]$$

$$E[(\hat{f} - f)^2] = E[(\hat{f} - E_D f + E_D f - f)^2] =$$

$$= \int (\hat{f}(\bar{x}) - E_D f(\bar{x}))^2 \, \underset{p(\bar{x})}{d\bar{x}} - 2 \cancel{\int (\hat{f} - E_D f)(E_D f - f) \, p(\bar{x}) \, d\bar{x}} + \int (E_D f - f)^2 \, p(\bar{x}) \, d\bar{x}$$

$$EPE[f] = E[(\hat{f}(\bar{x}) - E_D f(\bar{x}))^2] \qquad \boxed{\text{Bias}}$$

$$+ E[(E_D f(\bar{x}) - f(\bar{x}; D))^2] \qquad \text{Variance}$$

$$+ E[(y - \hat{f}(\bar{x}))^2] \qquad \text{Noise}$$