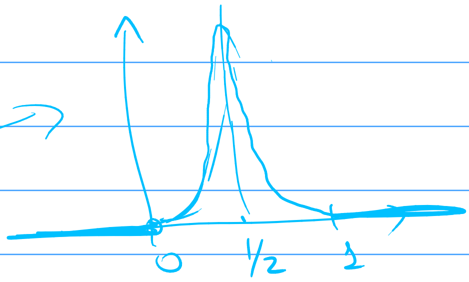
$$p(x|y) = \frac{p(x,y)}{p(y)}$$

$$p(y|x) = \frac{p(x,y)}{p(x)}$$

формула Баеса

$$\boxed{p(x|y) = \frac{p(x)\,p(y|x)}{p(y)}}$$

$$p(x,y) = p(x|y)p(y) = p(y|x)p(x)$$



параметры модели

данные

$$\underbrace{p(\theta|D)}_{\text{posterior}} = \frac{\overbrace{p(\theta)}^{\text{prior}} \cdot \overbrace{p(D|\theta)}^{\text{likelihood}}}{\underbrace{p(D)}_{\text{evidence}}}$$

$\theta^n(1-\theta)^m$

$\theta \to \max$

$$p(\theta|D) \propto p(\theta)p(D|\theta)$$

$\theta = p(\text{„решка"})$ $\quad D = \text{роорррор}$

$1-\theta = p(\text{„орёл"})$ $\quad \theta\cdot(1-\theta)\ldots$

$$p(D|\theta) = \theta^4(1-\theta)^3 \xrightarrow[\theta]{} \max$$

max likelihood

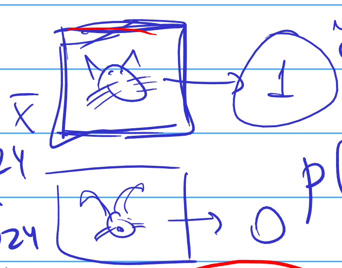
$$\theta_{ML} = \arg\max p(D|\theta) = 4/7$$

$$\frac{\partial p(D|\theta)}{\partial\theta} = 4\theta^3(1-\theta)^3 - 3\theta^4(1-\theta)^2$$

$$= \theta^3(1-\theta)^2(4(1-\theta)-3) = 0$$

$$\theta = 0, 1, 4/7$$

$D = \text{„орёл"}$ $\quad \theta_{ML} = 0$



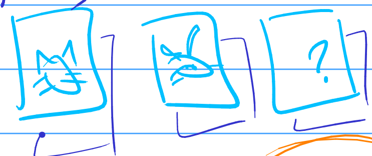
$1024 \times 1024 \times 3$

$p(y|\bar{x})$

input data

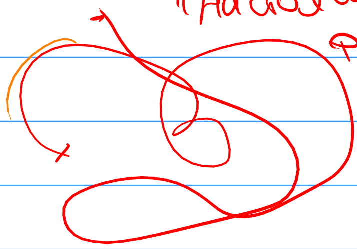
$$\bar{x} \in \mathbb{R}^{3\cdot 10^6}$$

$p(\bar{x})$



$$\mathbb{R}^{3\cdot 10^6} \rightsquigarrow \mathbb{R}^{2\cdot 10^3}$$

„настоящие фото"



$p(t|d) = 0.95$
$p(\bar{t}|d) = 0.05$

$$p(d|t) = \frac{\overset{0.95}{p(t|d)} \cdot \overset{0.01}{p(d)}}{p(t)} =$$

$$p(t) = p(t,d) + p(t,\bar{d}) =$$

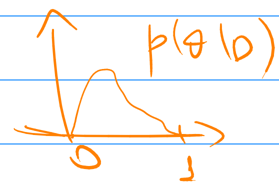
$$= p(t|d)p(d) + \underbrace{p(t|\bar{d})}_{0.05}\underbrace{p(\bar{d})}_{0.99}$$

$$= \frac{0.95 \cdot 0.01}{0.95 \cdot 0.01 + 0.05 \cdot 0.99} \approx 0.16$$

$$p(\theta|D) = \frac{p(D|\theta)\,p(\theta)}{p(D)}$$

1) Max likelihood θ_{ML}

$$p(D|\theta) \xrightarrow[\theta]{} \max$$



2) Posterior: $p(\theta|D) \propto p(\theta)\,p(D|\theta)$

Maximum a posteriori $\theta_{MAP} = \arg\max_{\theta} p(\theta|D)$

3) Predictive distribution $p(x|D) = \int p(x,\theta|D)\,d\theta =$

$$= \int \underbrace{p(x|\theta,D)}_{\substack{\| \\ p(x|\theta) \\ \text{likelihood}}} \underbrace{p(\theta|D)}_{\text{posterior}}\,d\theta$$

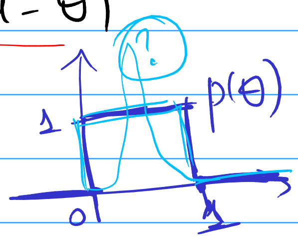
$$\boxed{p(x|D) = \int p(x|\theta)\,p(\theta|D)\,d\theta}$$

$$p(D|\theta) = \theta^n (1-\theta)^m$$

n – число решек
m – -//- орлов

$p(D|\theta) = \theta$

$\boxed{p(\theta)}$ prior



$$p(\theta) = \begin{cases} 1, & \theta \in [0,1] \\ 0, & \theta \notin [0,1] \end{cases}$$

$p(\theta|D) = ?$

$p(x|D) = ?$

$$p(\theta|D) \propto \underline{p(\theta)}\, \underline{p(D|\theta)} = \begin{cases} \theta^n(1-\theta)^m, & \theta \in [0,1] \\ 0, & \theta \notin [0,1] \end{cases}$$

$$\theta_{MAP} = \frac{n}{n+m} = \theta_{ML}$$

$$p(\theta|D) = \begin{cases} \frac{\theta^n(1-\theta)^m}{p(D)}, & \theta \in [0,1] \\ 0, & \underline{\theta \notin [0,1]} \end{cases}$$

$$p(D) = \int_0^1 \theta^n (1-\theta)^m \, d\theta =$$

$$= B(n+1, m+1) =$$

$$= \frac{n!\, m!}{(n+m+1)!}$$

$$\underline{B(\alpha, \beta)} =$$

$$= \int x^{\alpha-1}(1-x)^{\beta-1}\, dx$$

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$\Gamma(n+1) = n!$$

$$\underline{p(\theta|D) = \frac{(n+m+1)!}{n!\,m!} \cdot \theta^n(1-\theta)^m}$$

$$p(\text{решка}|D) = \int p(\text{решка}, \theta|D)\, d\theta =$$

$$= \int_{\mathbb{R}} \underbrace{p(\text{решка}|\theta, \cancel{D})}_{\theta} \boxed{p(\theta|D)}\, d\theta =$$

$$= \int_0^1 \theta \cdot \frac{(n+m+1)!}{n!\,m!} \theta^n (1-\theta)^m\, d\theta =$$

$$= \frac{(n+m+1)!}{n!\,m!} \underbrace{\int_0^1 \theta^{n+1}(1-\theta)^m\, d\theta}_{B(n+2,\, m+1)} = \frac{\cancel{(n+m+1)!}}{\cancel{n!}\,\cancel{m!}} \cdot \frac{(n+1)!\,\cancel{m!}}{(n+m+2)!}$$

Laplace's Rule

$$\boxed{p(\text{решка}|D) = \frac{n+1}{n+m+2}}$$