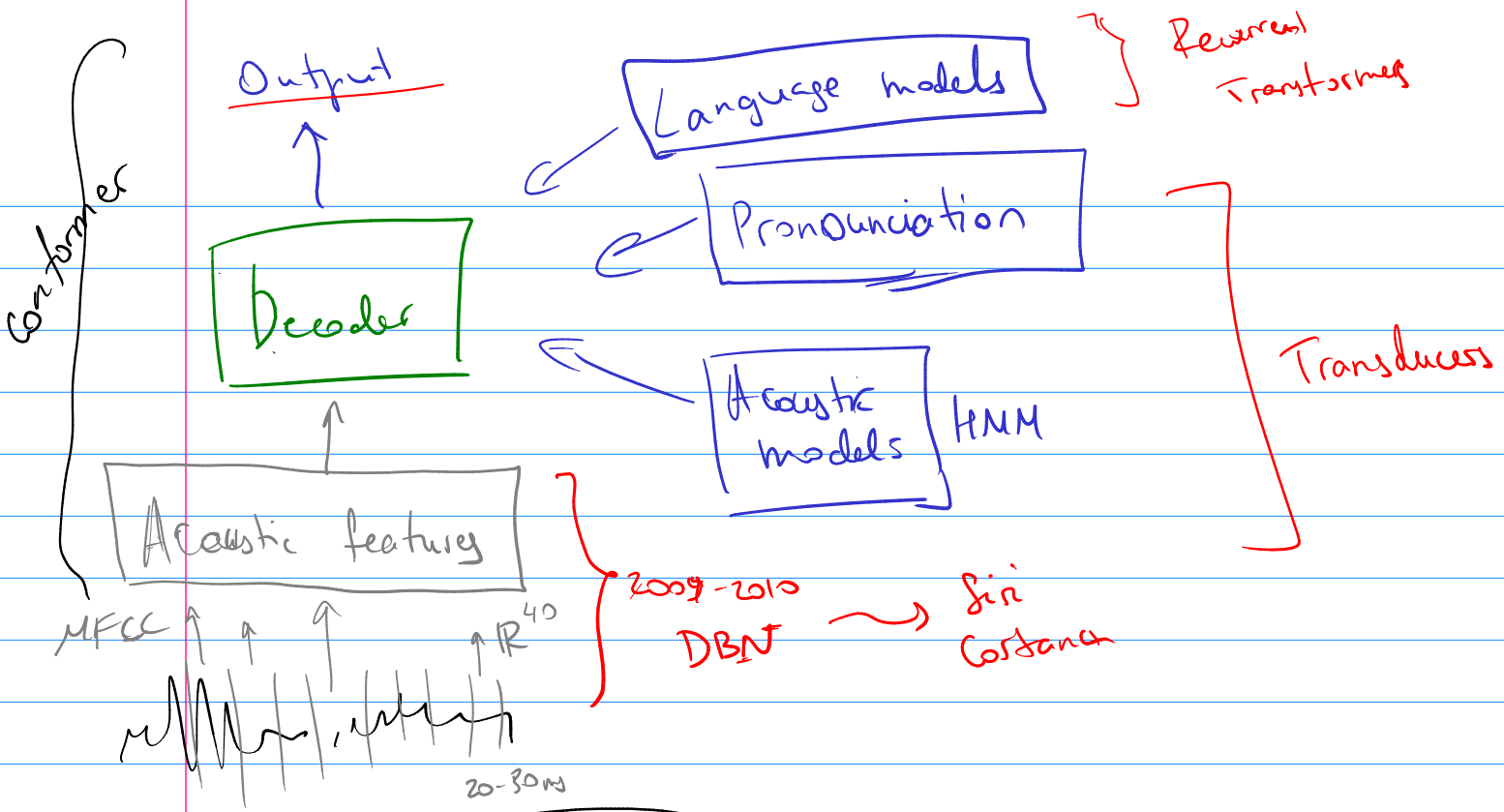


$p(y|\bar{x})$ disc.

$p(\bar{x}, y)$ generative

$$p(\bar{x}, y) = p(y|\bar{x}) p(\bar{x})$$

Naive Bayes

$\bar{x} = (x_1, \dots, x_k)$

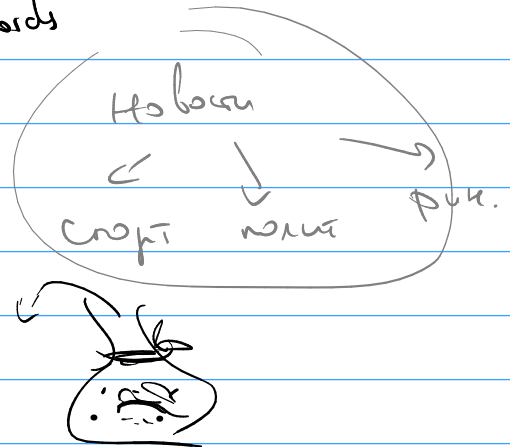
y

bag of words

$$p(\bar{x}|y) = \prod_{k=1}^K p(x_k|y)$$

generative

$$p(y, \bar{x}) = p(y) \cdot \prod_{k=1}^K p(x_k|y)$$



Logistic regression

$$p(y|\bar{x}) = \sigma(\bar{w}^T \bar{x}) = \frac{e^{\bar{w}^T \bar{x}}}{1 + e^{\bar{w}^T \bar{x}}} = \frac{1}{1 + e^{-\bar{w}^T \bar{x}}}$$

$$p(y|\bar{x}) = \frac{e^{\bar{w}^T \bar{x}}}{1 + e^{\bar{w}^T \bar{x}}}$$

$$p(\bar{y}|\bar{x}) = 1 - p(y|\bar{x}) = \frac{1}{1 + e^{\bar{w}^T \bar{x}}}$$

$$p(y|\bar{x}) \propto e^{\bar{w}^T \bar{x}}$$

$$p(\bar{y}|\bar{x}) \propto e^0$$

discriminative

$$p(y|\bar{x}) = \frac{1}{z(\bar{x})} e^{\bar{w}_y^T \bar{x}} = \frac{1}{z(\bar{x})} \left(\frac{w_{y,x_1}}{w_{y,x_2}} \right) \left(\frac{e^{w_{y,x_1} x_1}}{e^{w_{y,x_2} x_2}} \right)$$

$$p(y_k|\bar{x}) \propto e^{\bar{w}_k^T \bar{x}}$$

$$p(y_k|\bar{x}) = \frac{e^{\bar{w}_k^T \bar{x}}}{\sum \dots}$$

Generative-discr pair:

$$p(y, \bar{x}) = p(y) \prod_{i=1}^n p(x_i|y)$$

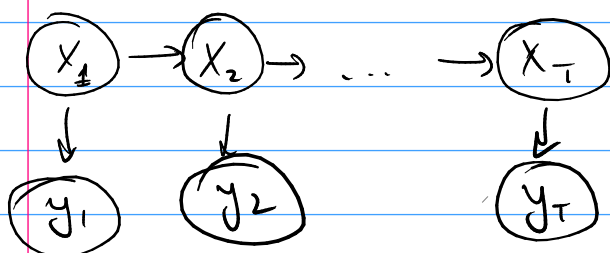
$$p(y|\bar{x}) = \frac{1}{z(\bar{x})} \cdot \prod_{i=1}^n f(y, x_i)$$

$$+ p(\bar{x})$$

$$p(\bar{x}, y) = p(\bar{x}) p(y|\bar{x})$$

Hidden Markov Models

generative



$$A = (a_{ij}) \quad a_{ij} = p(x_{t+1}=j | x_t=i)$$

$$B = (b_j(k)) \quad b_j(k) = p(y_{t+1}=k | x_{t+1}=j)$$

$$p(\bar{x}, \bar{y}) = p(x_1) p(y_1|x_1) p(x_2|x_1) p(y_2|x_2) \dots p(x_T|x_{T-1}) p(y_T|x_T)$$

$$p(\bar{x}, \bar{y}) = \frac{1}{z} \cdot e^{\sum_{i,j} \ln a_{ij} \cdot \left(\sum_t [x_t=i, x_{t+1}=j] \right) + \sum_{j,k} \ln b_j(k) \cdot \left(\sum_t [x_t=j, y_{t+1}=k] \right)}$$

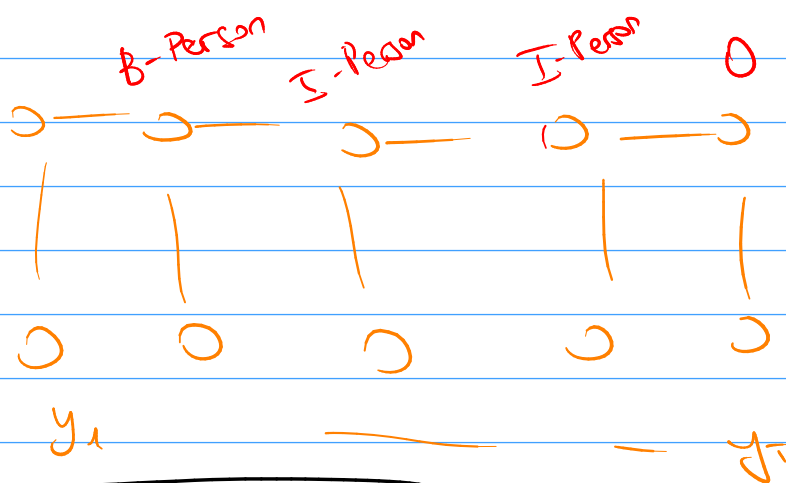
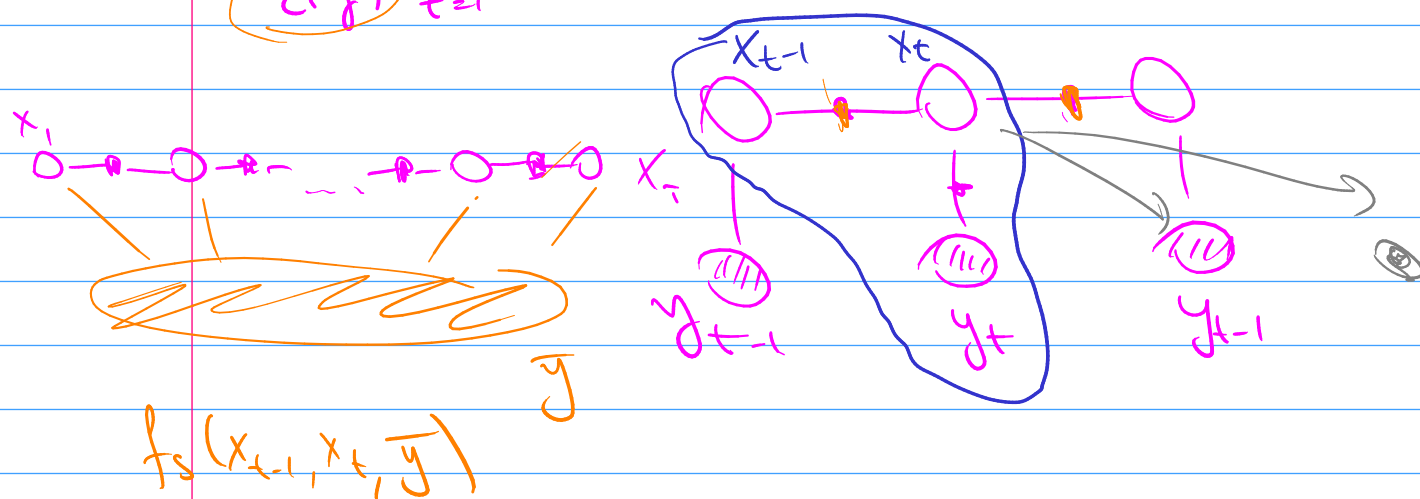
$$f(x_{1:t}, x_t, y_{t+1})$$

Conditional random field, CRF

linear chain CRF

$$p(\bar{y}|\bar{x}) = \frac{1}{Z} \cdot \prod_{t=1}^T e^{\sum_{s=1}^S \theta_s f_s(x_{t-1}, x_t, y_t)} \quad f_s = \begin{bmatrix} \dots \\ \dots \end{bmatrix} \quad \text{HMM}$$

$$p(\bar{x}|\bar{y}) = \frac{1}{Z(\bar{y})} \prod_{t=1}^T e^{\sum_{s=1}^S \theta_s f_s(x_{t-1}, x_t, y_t, y_{t+1}, y_{t+2})}$$



discr.

$$p(y|\bar{x})_{\bar{\theta}}$$

$$D = \{(\bar{x}, y)\}$$

$$= \prod p(\bar{x}|\bar{\theta})$$

$$\hat{p}(D|\bar{\theta}) \xrightarrow{\bar{\theta}} \max$$

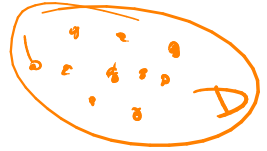
gen.

$$p(y, \bar{x}) \in \mathbb{R}^{3 \cdot 10^6} \approx p(y|\bar{x}) p(\bar{x})$$

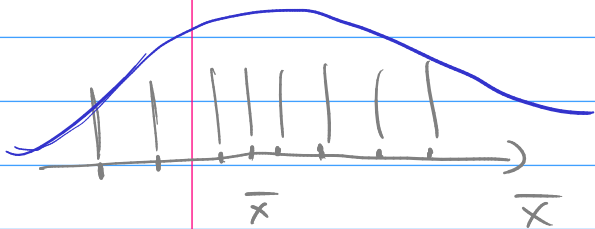
$$\bar{\theta}_{NL} = \operatorname{argmax}_{\bar{\theta}} \sum_{\bar{x} \in D} \log p(\bar{x}|\bar{\theta}) + \log p(\bar{\theta})$$

$$N = |D|$$

$$= \operatorname{argmin} \left(- \sum_{\bar{x} \in D} \underbrace{\left(\frac{1}{N} \right)}_{\parallel} \cdot \log \frac{p(\bar{x}|\theta)}{\underbrace{\left(\frac{1}{N} \right)}} \right) =$$



$$\boxed{KL(p||q) = - \int p \ln \frac{q}{p} dx}$$



$$= \operatorname{argmin} \left[KL(p_{data} || p_{model}) - \log p(\theta) \right]$$

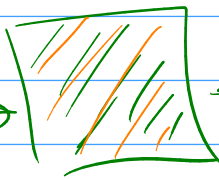
MAP:

$$\theta_{MAP} = \operatorname{argmax}_{\theta} p(\theta) p(D|\theta)$$

$$= \operatorname{argmin} \left[-\log p(D|\theta) - \log p(\theta) \right]$$

random noise

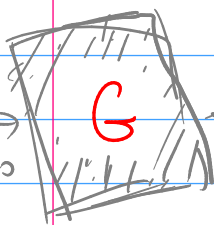
$\bar{z}$



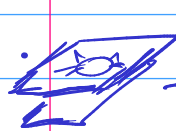
$p(\bar{x})$

$p(\bar{x}|\bar{c})$

$\bar{z} \in \mathbb{R}^{100}$

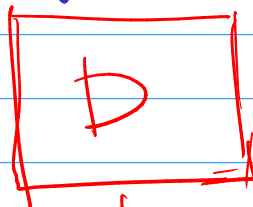


D:



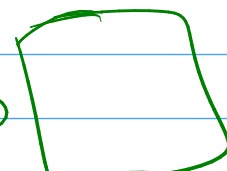
real

fake

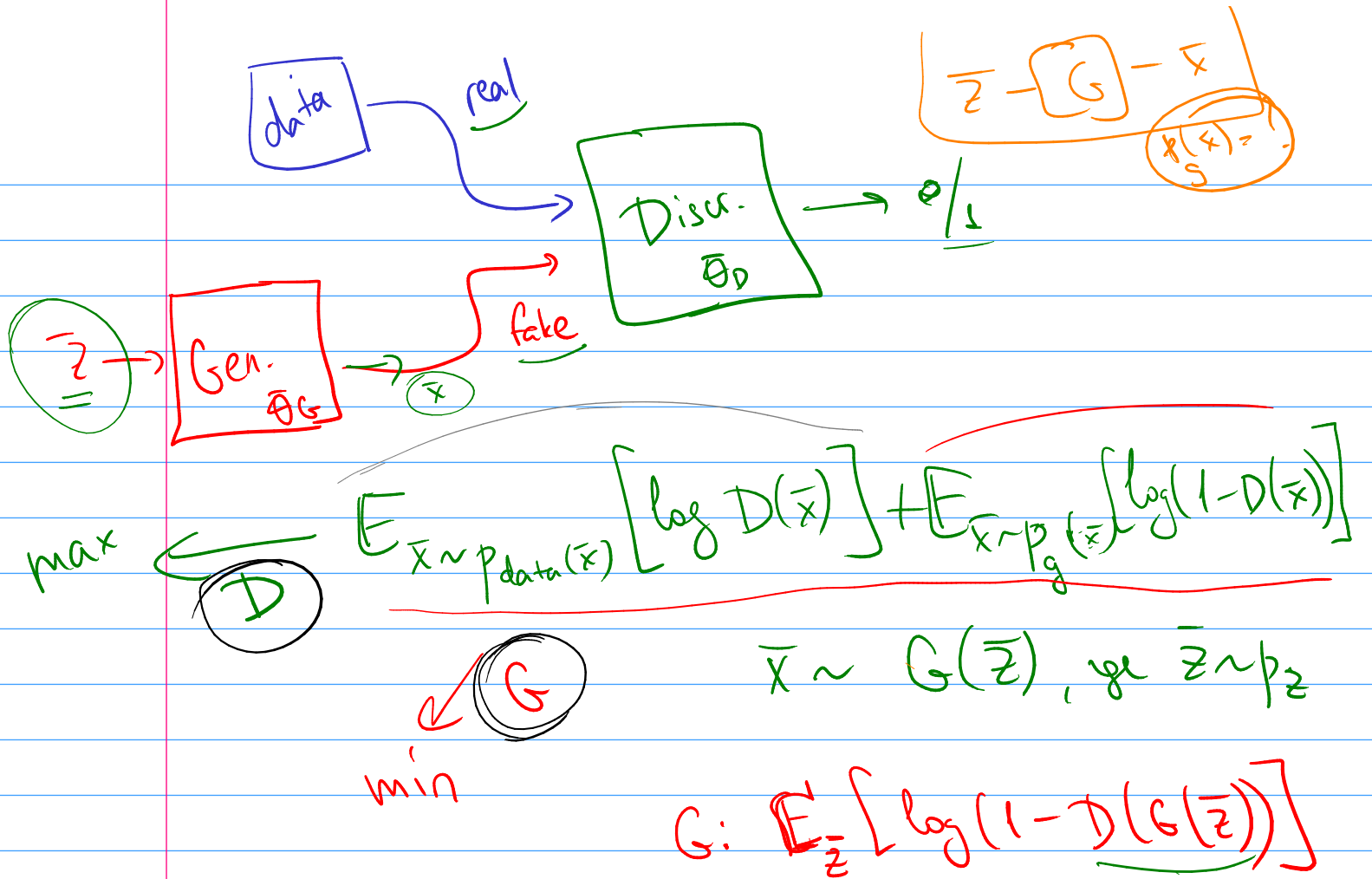


0/1 ?

$\bar{z}$



$\bar{c}$



GAN:

$$\min_G \max_D \left[\mathbb{E}_{\bar{x} \sim p_{\text{data}}} [\log D(\bar{x})] + \mathbb{E}_{\bar{z}} [\log(1 - D(G(\bar{z})))] \right]$$