# WORD EMBEDDINGS II: GLOVE AND EXTENSIONS 

Natural Language Processing

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GLOVE

## GLOVE

- GloVe - in a way, a variation of LSA.
- We are trying to approximate the cooccurrence matrix $X \in \mathbb{R}^{V \times V}$.
- Let $X_{i j}$ be how many times word $i$ cooccurs in our corpus with word $j, X_{i}=\sum_{j} X_{i j}$. Then

$$
p_{i j}=p(j \mid i)=\frac{X_{i j}}{X_{i}}=\frac{X_{i j}}{\sum_{k} X_{i k}} .
$$

i.e., $p_{i j}$ is the probability of the fact that word $j$ occurs in the context of word $i$.

- If we tried to approximate the matrix of $p_{i j}$, it would be almost exactly like LSA.


## GLOVE

- But we want to approximate the matrix of ratios $\frac{p_{i j}}{p_{k j}}$.
- The values $p_{i j}=p(j \mid i)$ themselves are hard to compare.
- But $p_{i k}$ and $p_{j k}$ for the same word $k$ do become comparable.
- Example from a 6 billion token corpus:

| Probability and Ratio | $k=$ solid | $k=$ gas | $k=$ water | $k=$ fashion |
| :--- | :---: | :---: | :---: | :---: |
| $P(k \mid$ ice $)$ | $1.9 \times 10^{-4}$ | $6.6 \times 10^{-5}$ | $3.0 \times 10^{-3}$ | $1.7 \times 10^{-5}$ |
| $P(k \mid$ steam $)$ | $2.2 \times 10^{-5}$ | $7.8 \times 10^{-4}$ | $2.2 \times 10^{-3}$ | $1.8 \times 10^{-5}$ |
| $P(k \mid$ ice $) / P(k \mid$ steam $)$ | 8.9 | $8.5 \times 10^{-2}$ | 1.36 | 0.96 |

## GLOVE

- We train the function

$$
F\left(\mathbf{w}_{i}, \mathbf{w}_{j} ; \tilde{\mathbf{w}}_{k}\right) \approx \frac{p_{i j}}{p_{j k}}
$$

where $\mathbf{w}_{i}$ and $\mathbf{w}_{j}$ are word vectors (embeddings) for words $i$ and $j$ in the space $\mathbb{R}^{d}$, and $\widetilde{\mathbf{w}}_{k}$ are context vectors that ensure that we approximate the ratio in the context of $k$.

- What is $F$ going to be?
- Theoretically, it could be a very complicated function, say, a deep neural network.
- But in reality we want the relations between word vectors to be simple (king-man+woman $\approx q u e e n)$.


## GLOVE

- So we train a simple function, assuming that

$$
F\left(\left(\mathbf{w}_{i}-\mathbf{w}_{j}\right)^{\top} \tilde{\mathbf{w}}_{k}\right)=\frac{F\left(\mathbf{w}_{i}^{\top} \tilde{\mathbf{w}}_{k}\right)}{F\left(\mathbf{w}_{j}^{\top} \tilde{\mathbf{w}}_{k}\right)}=\frac{p_{i j}}{p_{k j}} .
$$

This makes the model train simple relations between word vectors.

- And one more reasonable assumption: $F$ shouldn't change when we pass from $X$ to $X^{\top}$ and from w to $\tilde{\mathbf{w}}$.
- To add this symmetry, we assume that $F$ not only maps $\mathbf{w}_{i}-\mathbf{w}_{j}$ to the ratio of probabilities, but in general maps sums of arguments to products of function values:

$$
F\left(\left(\mathbf{w}_{i}-\mathbf{w}_{j}\right)^{\top} \tilde{\mathbf{w}}_{k}\right)=\frac{F\left(\mathbf{w}_{i}^{\top} \tilde{\mathbf{w}}_{k}\right)}{F\left(\mathbf{w}_{j}^{\top} \tilde{\mathbf{w}}_{k}\right)}=\frac{p_{i j}}{p_{j k}} .
$$

- What kind of a function is $F$ then?


## GLOVE

- $F$ actually has to be an exponent:

$$
\mathbf{w}_{i}^{\top} \tilde{\mathbf{w}}_{k}=\log \left(p_{i k}\right)=\log \left(X_{i k}\right)-\log \left(X_{i}\right)
$$

- We can hide $\log \left(X_{i}\right)$ in bias terms $\mathbf{b}_{i}$, getting a nice symmetric model:

$$
\mathbf{w}_{i}^{\top} \tilde{\mathbf{w}}_{k}+b_{i}+\tilde{b}_{k}=\log \left(X_{i k}\right)
$$

## GLOVE

- Two problems left:
- $\log \left(X_{i k}\right)$ very often diverges because $X_{i k}$ is often zero; generally, $X$ is a very sparse matrix;
- the model treats all $X_{i k}$ the same, but for rare words the ratio is very random, and for very frequent words it's not very important.
- In GloVe, we solve these problems by training wand $\tilde{\mathbf{w}}$ via weighted sum of squares loss function.


## GLOVE

- Thus, the objective function for GloVe will be

$$
J=\sum_{i, j=1}^{V} f\left(X_{i j}\right)\left(\mathbf{w}_{i}^{\top} \tilde{\mathbf{w}}_{j}+b_{i}+\tilde{b}_{j}-\log X_{i j}\right)^{2},
$$

where $f$ is a nondecreasing function with $f(0)=0$ that doesn't grow too fast, e.g.,

$$
f(x)= \begin{cases}\left(\frac{x}{x_{\max }}\right)^{\alpha}, & \text { if } x<x_{\max } \\ 1 & \text { otherwise }\end{cases}
$$

- Demo: nearest neighbors, geometric relations.


## EXTENSIONS

- Some modifications of word embeddings add external information.
- (Levy et al., 2014): use dependency parsing for local context.

- The RC-NET model (Xu et al. 2014) extends skip-grams with relations (semantic and syntactic) and categorical knowledge (sets of synonyms, domain knowledge etc.).

- We would like to add relations from Freebase or similar knowledge bases - how?
- The basic word2vec model gets a regularizer for every relation that tries to bring it closer to a linear relation between the vectors, so that, e.g.,

$$
\mathbf{w}_{\text {Hinton }}-\mathbf{w}_{\text {Wimbledon }} \approx r_{\text {born at }} \approx \mathbf{w}_{\text {Euler }}-\mathbf{w}_{\text {Basel }}
$$



## WORD SENSE DISAMBIGUATION

- Another important problem with both word vectors and char-level models: homonyms.
- How do we distinguish different senses of the same word?
- the model usually just chooses one meaning;
- e.g., let's check nearest neighbors for words like converse, jaguar, and other homonyms.
- We have to add latent variables for different meaning and infer them from context.


## WORD SENSE DISAMBIGUATION

- To train the meanings with latent variables - Bayesian inference with stochastic variational inference (Bartunov et al., 2015).
- Problem: we don't know in advance how many senses a word has.
- Basic idea - set a prior distribution that allows for any number of senses, just with decreasing probabilities.
- Stick-breaking priors on the senses $z_{w}$ :
$p(z=k \mid w, \beta)=\beta_{w k} \prod_{r=1}^{k-1}\left(1-\beta_{w r}\right), \quad p\left(\beta_{w k} \mid \alpha\right)=\operatorname{Beta}\left(\beta_{w k} \mid 1, \alpha\right)$.


## WORD SENSE DISAMBIGUATION

- The total likelihood is now

$$
\begin{aligned}
& p(C, Z, \beta \mid W, \alpha, \theta)= \\
& \quad=\prod_{w=1}^{V} \prod_{k=1}^{\infty} p\left(\beta_{w k} \mid \alpha\right) \prod_{i=1}^{N} p\left(z_{i} \mid w_{i}, \beta\right) \prod_{j=1}^{N} p\left(c_{i j} \mid z_{i}, w_{i}, \theta\right)
\end{aligned}
$$

- And we are optimizing

$$
p(C \mid W, \alpha, \theta)=\int \sum_{Z} p(C, Z, \beta \mid W, \alpha, \theta) \mathrm{d} \beta .
$$

- Hard but possible to optimize - stochastic variational inference.


## WORD SENSE DISAMBIGUATION

- Nice results:

| ALPHA | "LIGHT" <br> nearest neighbours |  |  | $p(z)$ |
| :---: | :---: | :--- | :--- | :--- |
| Skip-gram | 1.00 | far-red, emitting | "CORE" <br> nearest neighbours |  |
|  | 1.00 | far-red, illumination | 1.00 | cores, component, i7 |
|  |  |  | 0.40 | corium, cores, sub-critical |
| 0.075 | 0.28 | armoured, amx-13, kilcrease | 0.60 | basic, i7, standards-based |
|  | 0.72 | bright, sunlight, luminous | 0.34 | competencies, curriculum |
|  | 0.09 | tvärbanan, hudson-bergen | 0.36 | nucleus, backbone |
|  | 0.17 | dark, bright, green | 0.21 | reactor, hydrogen-rich |
|  | 0.09 | 4th, dragoons, 2nd | 0.13 | intel, processors |
| 0.1 | 0.26 | radiation, ultraviolet | 0.27 | curricular, competencies |
|  | 0.28 | darkness, shining, shadows | 0.15 | downtown, cores, center |
|  | 0.11 | self-propelled, armored | 0.24 | nucleus, rag-tag, roster |

- The hyperparameter $\alpha$ controls how many senses are probable:



Thank you for your attention!

