$$\sum_\theta p(\theta|D)\,p(x|\theta)$$

$$p(D|\theta) \xrightarrow{\theta_{ML}} \max \qquad p(\theta|D) \propto p(D|\theta)p(\theta) \xrightarrow{\theta_{MAP}} \max \qquad p(x|D) \propto \int p(\theta|D)\,p(x|\theta)\,d\theta$$

маргинализация marginal

$$p(\bar x) = p(x_1, x_2, \dots, x_n)$$

$$p(x_1|x_2 \dots x_n) = \frac{p(x_1 \dots x_n)}{p(x_2 \dots x_n)}$$

$$p(x_2 \dots x_n) = \int p(\bar x)\,dx_1 = p(0, \dots) + p(1, \dots)$$

$$p(x_1) = \int p(\bar x)\,dx_2 \dots dx_n \qquad \sum_{x_2} \dots \sum_{x_n} p(x_1 \dots x_n)$$

$$p(\theta, x|D) \qquad p(x_n|\theta) \qquad p(\theta, x, D) \qquad \int p(\theta, x|D)\,d\theta$$

$$p(\bar x) \qquad p(x_k) = \sum_{x_1..x_k..x_n} p(x_1 \dots x_k \dots x_n)$$

$$= \left[p(x_1)p(x_2|x_1)p(x_3|x_2) \dots p(x_n|x_{n-1}) \right]$$

$\bar x$ — $\boxed{p}$ — $p(\bar x)$

$10..0 \qquad p(1, 0..0)$

$$p(x_1 \dots x_n, y_1 \dots y_m) = p(x_1 \dots x_n)\,p(y_1 \dots y_m)$$

$$p(X) = \prod_{X_i} p(X_i) \qquad X_i \subseteq X$$

$$p(x_k) - \sum_{X \setminus x_k} p(X)$$

$$\boxed{p(\bar w, D) = p(\bar w)\,p(D|\bar w)}$$

$$p(\bar w|D) = \frac{p(\bar w, D)}{p(D)} \qquad p(D) = \int p(\bar w, D)\,d\bar w$$

$$\boxed{p(D|\bar w) = \prod_n p(d_n|\bar w)}$$

условная независимость

$p(\bar w, D)$ — d_n, d_m усл. нез. при усл. $\bar w$

conditional independence

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n=1: x — $p(x)$

n=2: $p(x,y) = p(x)p(y)$ — x y

$$p(x,y) = p(x)p(y|x) = p(y)p(x|y)$$

$x \to y \qquad x \leftarrow y$

$$p(\bar x) = \prod_{i=1}^n p(x_i|\mathrm{Par}(x_i))$$

n=3:

$$p(x,y,z) = p(x)p(y)p(z) \qquad x \quad y \quad z$$

$$p(x,y,z) = p(x)p(y)p(z|y) \qquad x \quad y \to z$$

$$p(x,y,z) = p(x)p(y|x)p(z|x,y)$$

tournaments

$$p(z)p(y|z) \times p(x|y,z)$$

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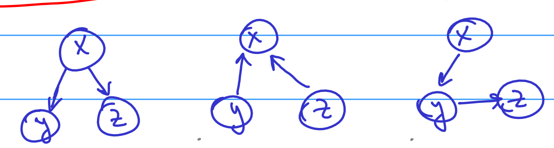
n=3

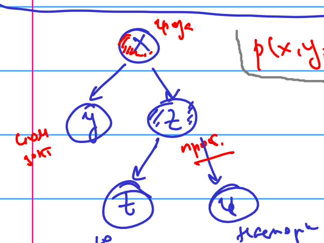


$$\boxed{p(x,y,z) = p(x)p(y|x)p(z|y)}$$

$$p(x,z|y) = p(x|y)p(z|y)$$

$$\frac{p(x,y,z)}{p(y)} = \frac{p(x)p(y|x)p(z|y)}{p(y)}$$

$p(x_1)$, $p(x_2|x_1)$, $p(x_3|x_1, x_2)$, $p(x_n|x_1 \dots x_{n-1})$



$$p(x,y,z) = p(x)p(y|x)p(z|x)$$

$$p(y,z|x) = p(y|x)p(z|x)$$

$$\frac{p(x,y,z)}{p(x)} = \frac{\cancel{p(x)}\,p(y|x)p(z|x)}{\cancel{p(x)}}$$

directed graphical models

Bayesian belief networks

$$p(\bar x) = \prod_i p(x_i|X_i) \qquad X_i = \mathrm{Par}(x_i)$$

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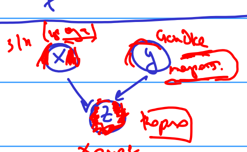
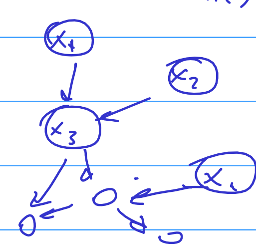
explaining away

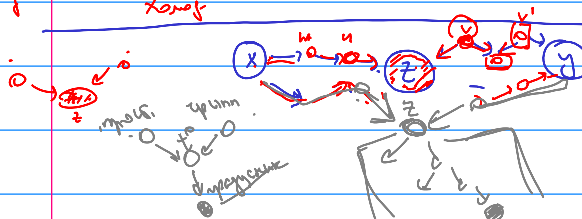


$$p(x,y,z) = p(x)p(y)p(z|x,y) \ne p(x,y)p(z|x,y)$$

$$p(x,y) = p(x)p(y)$$

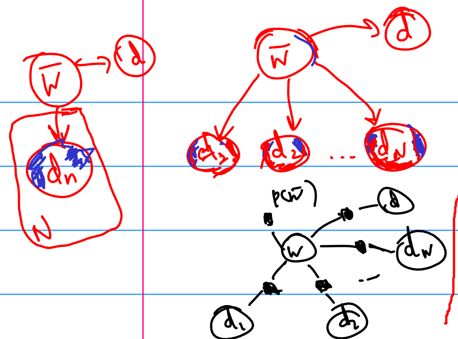
$$p(x,y|z) \ne p(x|z)p(y|z)$$





$$p(x,y|z) = p(x|z)p(y|z)$$

Лин. регр.



$$p(\bar w, D) = p(\bar w)\cdot \prod_n p(d_n|\bar w)$$

$$p(\bar w|D) = \frac{p(\bar w, D)}{p(D)}, \quad p(D) = \int p(\bar w, D)\, d\bar w$$

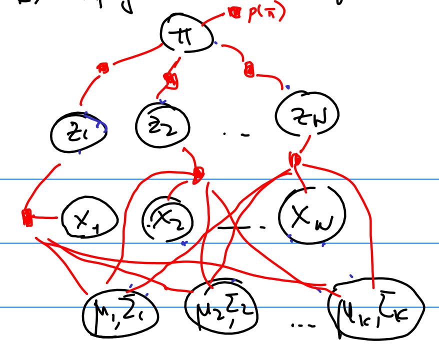
$$p(d|D) = \int p(d|\bar w)\, p(\bar w|D)\, d\bar w$$

$p(d, \bar w|D)$

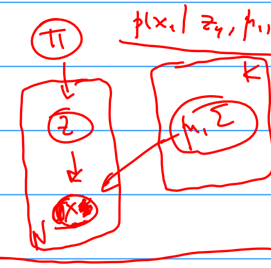
$$p(x|\dots) = \sum_{k=1}^{K} \pi_k\, p(x|\mu_k, \Sigma_k)$$

$$p(x, \boxed{z}|\dots) = \prod_k \left[\pi_k\, p(x|\mu_k, \Sigma_k)\right]^{z_k}$$

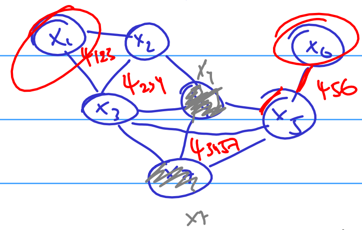
Кластеризация EM-алгоритм



$p(x_1| z_1, \mu_1, \Sigma_1 \dots \mu_k, \Sigma_k)$



Undirected graphical models

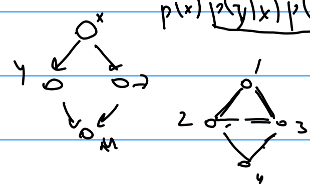


$$p(x_1 \dots x_7) = \psi_{123}(x_1, x_2, x_3)\, \psi_{234}\, \psi_{56}\, \psi_{3457}$$

ψ_1 ψ_{12} ψ_{13}

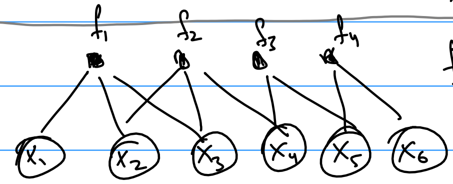
$p(x)\, p(y|x)\, p(t|x)$ ψ_{234}

$p(n|y, t)$



$$f = \psi_{123}\cdot \psi_{234}$$

Фактор-граф



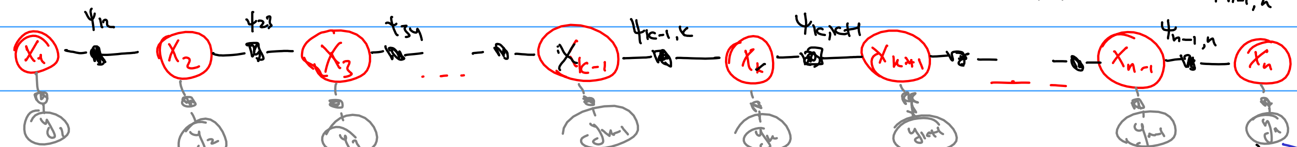
$$f(x_1 \dots x_6) = f_1(x_1, x_2, x_3)\, f_2(x_2, x_3, x_4)\, f_3(x_4, x_5)\, f_4(x_4, x_6)$$

$$f(X) = \psi_{12}\,\psi_{23}\dots\psi_{n-1,n}$$

$$p(x_1 \dots x_n) = \underbrace{p(x_1)\,p(x_2|x_1)}_{\psi_{12}}\,\underbrace{p(x_3|x_2)}_{\psi_{23}}\ \psi_{34} \dots \underbrace{p(x_n|x_{n-1})}_{\psi_{n-1,n}}$$

NLP

topic modeling



$$f(x_k) = \sum_{x_1 \dots \cancel{x_k} \dots x_n} f(x_1 \dots x_k, \dots, x_n) = \sum_{x_1 \dots x_n} \psi_{12}(x_1, x_2)\,\psi_{23}(x_2, x_3) \dots \psi_{k-1,k}(x_{k-1}, x_k)\,\psi_{k,k+1}(x_k, x_{k+1}) \dots \psi_{n-1,n}(x_{n-1}, x_n)$$

$$= \sum_{x_2}\sum_{x_3} \dots \sum_{x_{k-1}}\sum_{x_{k+1}} \dots \sum_{x_n}\left[\psi_{12}\,\psi_{23} \dots \psi_{n-1,n}\right]$$

$$\underbrace{\sum_{x_1}\sum_{x_2}\sum_{x_4}}_{8}\left[\psi_{12}(x_1, x_2)\,\psi_{23}(x_2, x_3)\,\psi_{34}(x_3, x_4)\right] = \sum_{x_1}\sum_{x_2}\left(\psi_{12}\,\psi_{23}\left(\sum_{x_4}\psi_{34}\right)\right) = \left[\sum_{x_1}\sum_{x_2}\psi_{12}\,\psi_{23}\right]\left[\sum_{x_4}\psi_{34}\right]$$

6

$$= \sum_{x_2}\left[\psi_{23}\sum_{x_1}\left[\psi_{12}\right]\right]\left[\sum_{x_4}\psi_{34}\right]$$

$u(x_3)\cdot v(x_3)$

$2(n-1)$

$$\sum_{x_1 \dots x_n}$$

2^{n-1}

$$\sum f(x_1 \dots x_n) = \underbrace{\sum_{x_1}\sum_{x_2} \dots \sum_{x_{k-1}}}\ \underbrace{\sum_{x_{k+1}} \dots \sum_{x_n}}\left[\underbrace{\psi_{12} \dots \psi_{k-1,k}}_{x_1 \dots x_k}\ \underbrace{\psi_{k,k+1} \dots \psi_{n-1,n}}_{x_k \dots x_n}\right] = \left[\sum_{x_1}\sum_{x_2} \dots \sum_{x_{k-1}} \psi_{12} \dots \psi_{k-1,k}\right]\left[\sum_{x_{k+1}} \dots \sum_{x_n} \psi_{k,k+1} \dots \psi_{n-1,n}\right]$$

2^{n-1}

$$= \left[\sum_{x_{k-1}} \psi_{k-1,k}\left(\sum_{x_{k-2}} \psi_{k-2,k-1}\left(\sum_{x_{k-3}} \dots \sum_{x_3} \psi_{3,4} \sum_{x_2} \psi_{23} \sum_{x_1} \psi_{12}\right) \dots\right)\right] \times$$

$f(x_2)$ $f(x_3)$ $f(x_k)$

$2(n-1)$

$$\times \left[\sum_{x_{k+1}} \psi_{k,k+1}\left[\sum_{x_{k+2}} \psi_{k+1,k+2}\left[\sum_{x_{k+3}} \dots \sum_{x_{n-1}} \psi_{n-2,n-1} \sum_{x_n} \psi_{n-1,n}\right] \dots\right]\right]$$

$$\sum_y f(x, y)$$

$$g(x) = \int f(x, y)\, dy$$

$$\dots \int u(x)^2 \dots$$