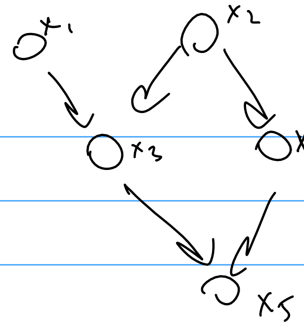
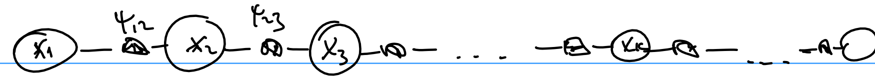


$$f(x_1) = \sum_{x_2 \ldots x_n} f(x_1 \ldots x_n)$$



$$f(x_1 \ldots x_n) = \psi_{12}\psi_{23} \ldots \psi_{n-1,n}$$

$$\sum_{\ldots \cancel{x_k} \ldots} \psi_{12}\psi_{23}\ldots\psi_{n-1,n} = \left[\sum_{x_1 \ldots x_{k-1}} \psi_{12} \ldots \psi_{k-1,k}\right]\left[\sum_{x_{k+1} \ldots x_n} \psi_{k,k+1} \ldots \psi_{n-1,n}\right]$$

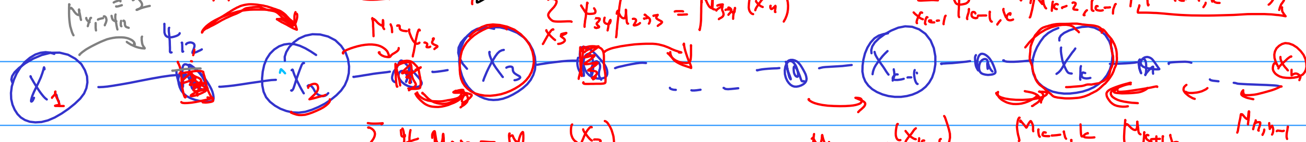
$$= \left[\sum_{x_{k-1}} \psi_{k-1,k} \sum_{x_{k-2}} \psi_{k-2,k-1} \left[\ldots \sum_{x_2} \psi_{23} \sum_{x_1} \psi_{12}\right] \ldots\right]\left[\sum_{x_{k+1}} \ldots \sum_{x_n}\right]$$

$$\sum_{x_1} \psi_{12} = \mu_{1\to 2}(x_2)$$

$$\mu_{y_1 \to \psi_{12}} = 1$$

$$\sum_{x_{k-1}} \psi_{k-1,k} \cdot \mu_{k-2,k-1} = \mu_{k-1,k}(x_k)$$

$$\sum_{x_3} \psi_{34}\mu_{2\to 3} = \mu_{3\to 4}(x_4)$$



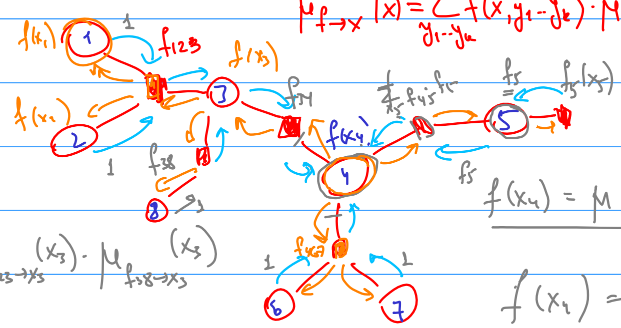
$$\sum_{x_2} \psi_{23}\mu_{1\to 2} = \mu_{2\to 3}(x_3)$$

$$\mu_{k-2,k-1}(x_{k-1}) \quad \mu_{k-1,k}(x_k) \quad \mu_{k+1,k}(x_k) \quad \mu_{n,n-1}$$

$$\mu_{f_{123}\to x_3}(x_3) = \sum_{x_1 x_2} f_{123} \cdot \mu_{x_1 \to f_{123}} \cdot \mu_{x_2 \to f_{123}}$$

$$\mu_{f\to x}(x) = \sum_{y_1 \ldots y_k} f(x, y_1 \ldots y_k) \cdot \mu_{y_1\to f}(y_1)\,\mu_{y_2\to f}(y_2) \ldots \mu_{y_k\to f}(y_k)$$

$$f(x_k) = \mu_{k-1,k} \cdot \mu_{k+1,k}$$



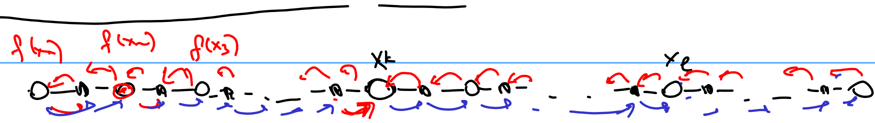
$$\mu_{x \to f}(x) = \mu_{g_1\to x}(x) \ldots \mu_{g_e \to x}(x)$$

$$f(x_4) = \mu \cdot \mu \cdot \mu$$

$$\mu_{x_3 \to f_{34}}(x_3) = \mu_{f_{123}\to x_3}(x_3) \cdot \mu_{f_{38}\to x_3}(x_3)$$

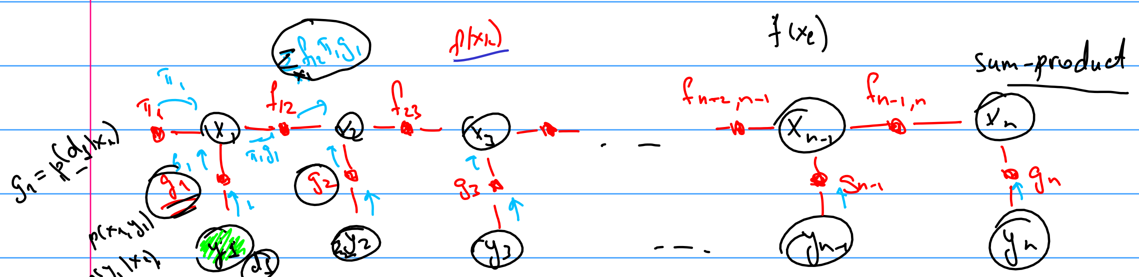
$$f(x_4) = \sum f(x_1 \ldots x_8) = \left[\sum \ldots\right]\left[\sum \ldots\right]\left[\sum \ldots\right]$$

$$\sum \ldots \left(\sum \ldots [\sum][\sum \ldots]\right)$$



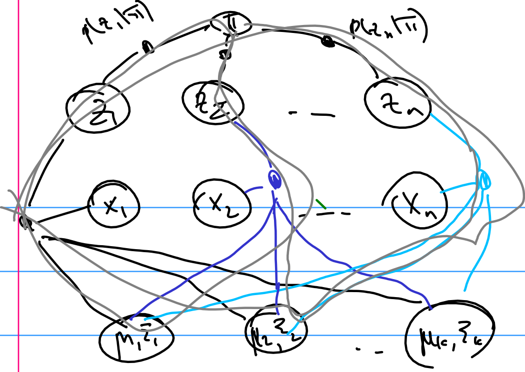
sum-product

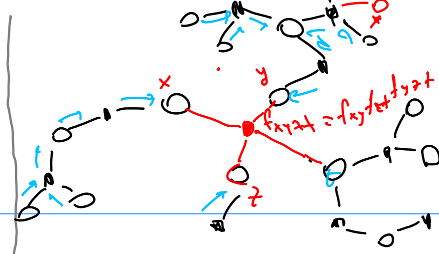
α, β, γ



$$p(x_1 \ldots x_n, y_1 \ldots y_n) = p(x_1)\,p(y_1|x_1)\,p(x_2|x_1)\,p(y_2|x_2) \ldots p(y_{n-1}|x_{n-1})\,p(x_n|x_{n-1})\,p(y_n|x_n)$$

$$f(x_1 \ldots x_n, y_1 \ldots y_n) = f_1(x_1) \cdot g_1(x_1, y_1) \cdot f_{12} \cdot g_2 \cdot \ldots g_{n-1} \cdot f_{n-1,n} \cdot g_n$$

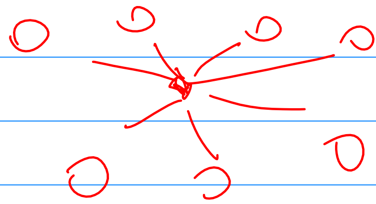


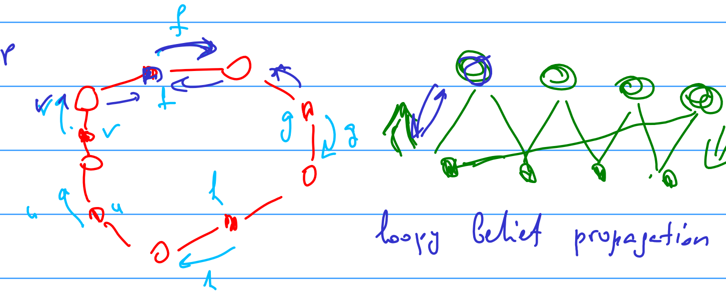


$f(X) = \prod \ldots f_{xy} f_{xt} f_{yz} \ldots$

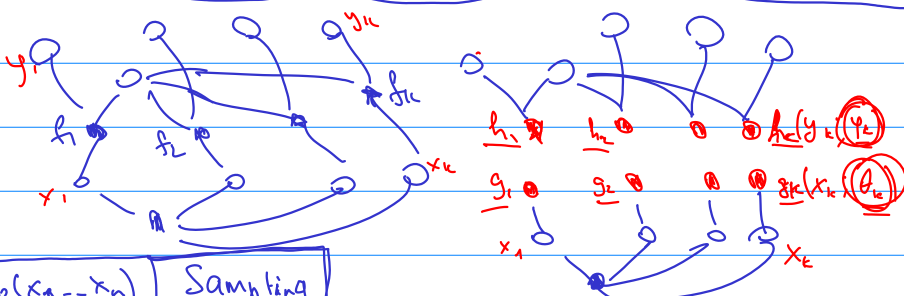
f_{xyzt}

$\sum_{xyt} f_{xyzt}$





loopy belief propagation



$F = f_1 f_2 \ldots f_k \cdot a \ldots$

$F \approx h_1 h_2 \ldots h_K g_1 g_2 \ldots g_k \, a \ldots$

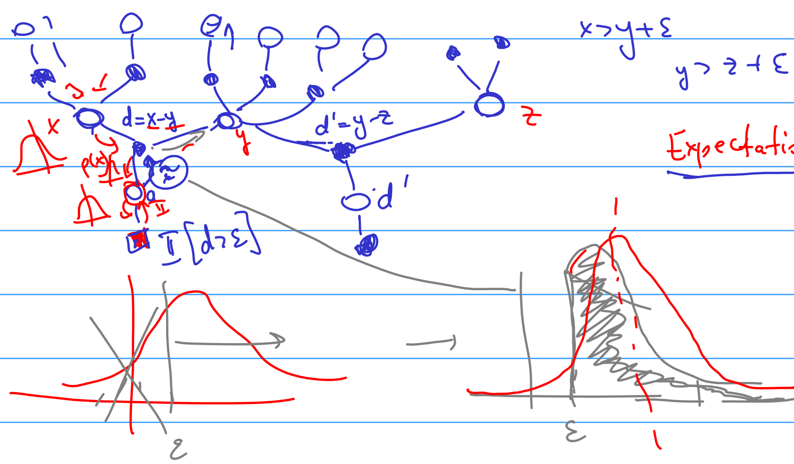
Variational approximation

$x_1 .. x_n \sim p(x_1 .. x_n)$	Sampling

x

y

z



$x > y + \varepsilon$

$y > z + \varepsilon$

Expectation Propagation

Factor graphs:

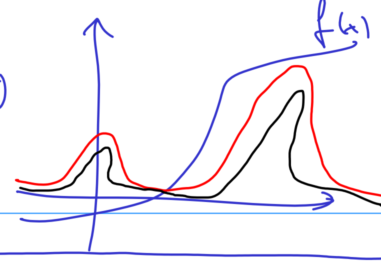
- message passing
- loopy BP
- variational approximation
- EP

$\int p(\bar{x})\, dx_2 \dots dx_n$

$\bar{x} \sim p(\bar{x})$

$p(x|D) = \int p(x|\bar\theta)\, p(\bar\theta|D)\, d\bar\theta = \mathbb{E}_{\bar\theta|D}\, p(x|\bar\theta)$

$\mathbb{E}_{\bar x \sim p(\bar x)} f(\bar x) = \frac{1}{R} \sum_{z=1}^{R} f(\bar x_z)$, $\bar x_z \sim p(\bar x)$



rand() ~ Unif([0,1])

$\bar x$ — $\boxed{p}$ — $p(\bar x) = 0.9$

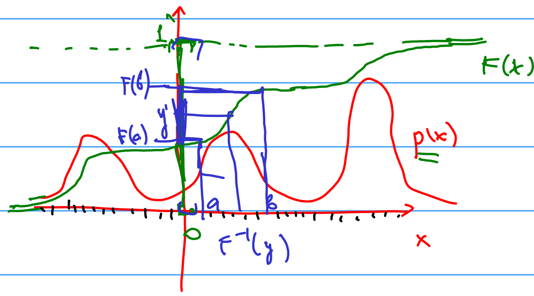
$\bar x$ — $\boxed{p^*}$ — $p^*(\bar x) \propto p(\bar x)$

$p(\bar\theta|D) = \frac{p(\bar\theta)\, p(D|\bar\theta)}{Z = \int p(\bar\theta)\, p(D|\bar\theta)\, d\bar\theta} \propto p(\bar\theta)\, p(D|\bar\theta) = p^*(\bar\theta|D)$

$x \sim U([0,1]) \Rightarrow y \sim U([-1,1])$

$y = 2x - 1$

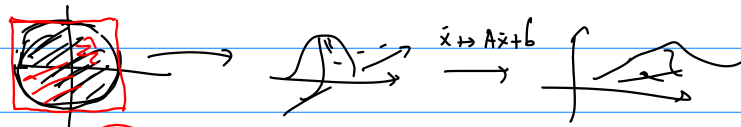
$x \sim U([0,1])$ $\qquad y \sim p(x) \in \mathbb{R}$

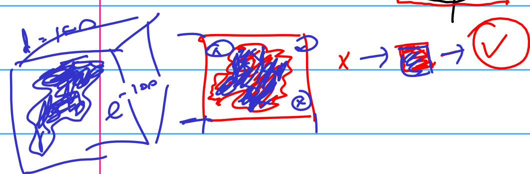


$p(x \in [a,b]) = \int_a^b p(x)\, dx = F(b) - F(a)$

- Box-Müller

$U([0,1])$





$p([x, x+dx]) = p(x)\, dx$