On topological works of N.Shanin

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Right now I am in awkward situation since I was asked to tell about topological papers of N.Shanin but it is known to me that N.Shanin himself consider them as something like "the original sin" in view of using of nonconstructive objects and reasoning. But all the same I dare to begin my talk since up the present I myself "live in sin".

Topological papers of N.Shanin (1943–1948) can be divided into two cycles. The first one (1943) containes papers on the theory of extensions of topological spaces and the second cycle (1946–1948) was devoted to study of products of topological spaces and the theory of diadic bicompacts.

Let us begin from the first cycle, which contains three papers published in DAN (Doklady Akademy Nauk). Here N.Shanin has considered two special types of extensions, which proved to be natural generalizations of Wallmen extensions. Wallman extensions of topological spaces can be defined as follows. Let X be a topological space and F be the set of all its closed sets. A system γ of nonempty closed sets is called *centred* one if all sets of any its finite subsystem have nonempty intersection. A centered system is called *vanishing* if the intersection of all its elements is the empty set. We can consider the set Γ of all maximal vanishing centred systems (*ultrafilters*) of closed sets of the space X and the union $X' = X \cup \Gamma$ of X and Γ (here X designates also the set of all points of the topological space X). So $X \subset X'$. Now, let us define the topological structure on X' such that sets $F' = F \cup \{\gamma | \gamma \in \Gamma\}$ are basic closed sets of the corresponding topological space X'. It is easy to prove that X' will be bicompact extension of X. It is the well known *Wallman's extension*. Generalizing this construction N.Shanin proposed firstly to consider not the whole system F of closed in X sets but various systems F^* of closed in X sets and after that to consider for any of these systems the corresponding set Γ^* of all maximal vanishing centred systems of its elements. These procedures gives us different bicompact extensions of any given topological space X – the *wallman type* extensions. Some later the hypothesis arose that any bicompact extension is the wallman type extension, but in the end it was proved that this hypothesis is incorrect. In the modern extension theory any bicompact extension is defined by some contiguity relation and some of such contiguity relations are defined by above mentioned Shanin's systems F^*.

N.Shanin considered also other extensions which can be obtained by the more strong wallman type construction. Here the condition of centredness of system is substituted by strong centredness: the intersection of *interiors* of sets must be nonempty. In this cycle of papers N.Shanin investigated various properties of introduced by him extensions: coincidence of extensions, comservation of weigh, separation properties, internal characteristic. The papers have played important part in the general extension theory.

Now, let us pass to the second cycle. It contains four papers published in DAN (1946) and the paper published in Trudah Matematicheskogo Instituta im. V.A.Steklova AN SSSR, the latter includes the others and so we shall consider this paper only. The title of the paper is "On product of topological spaces", it contains new notions and results on the theory of products of topological spaces, but the main results concern diadic bicompacts. Just these N.Shanin's results has been distinguished in the book "Mathematics in the SSSR during thirty years".

We shall not touch here Shanin's theorems of function splitting which play important but technical role. Let us begin with two fundamental notions introduced by N.Shanin, namely of *arranged in order families of sets* and of *gauge of topological spaces.*

A family A is called *ascending arranged in order* if $A_1 < A_2$, $A_1 \in A$, $A_2 \in A$ implies $A_1 \subset A_2$, a family A is called *descending arranged in order* if $A_1 < A_2$, $A_1 \in A$, $A_2 \in A$ implies $A_2 \subset A_1$.

Let X be a topological space and G be the family of all open nonempty sets of X. A cardinal number $\mu > 1$ is called *gauge of* X if for any subfamily A of G having the power μ there exists a nonvanishing family $A^* \subset A$ having the same power.

Any topological space has infinitly many gauges. N.Shanin has introduced different modifications of the notion of gauge to formulate his theorems but we shall not go into details and so in our formulating some (essential) details will be omited. For example he proved to be that if μ is a gauge of some topological spaces then μ is the gauge of their product, that μ will be a gauge of $\prod X_\xi$ taken for all $\xi \in \Xi$ if some simple conditions concerning μ and concerning topological spaces X_ξ are fulfilled, that μ will be a gauge of diadic bicompact if some condition concerning μ is fulfilled and so on. N.Shanin established also some connections between gauges and powers of arranged in order families of all open sets and anticentred families of such sets.

P,S.Alexandrov had proved that any compact metric space is a continuous image of $D^{\aleph_0}$ (Kantor's discontinuum). He had introduced also the general notion of diadic bicompact as a continuous image of D^μ where μ has an arbitrary power (generalized Kantor's discontinuum) and stated some hypotheses concerning diadic bicompacts. Shpilrain had proved that there are bicompacts which are not diadic bicompacts. The above mention N.Shanin's results and some additional results which are contained in three supplements to the basic paper had allowed him to prove many important statements of the theory of diadic bicompacts and, in particular, to solve some problems of the theory of diadic bicompacts stated by P.S.Alexandrov - the prominent contribution to the theory of diadic bicompacts.