Algorithms for Nearest Neighbors

State-of-the-Art

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Outline

1. Problem Statement
   - Applications
   - Data Models
   - Variations of the Computation Task

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2. Overview of Algorithmic Techniques
   - Partitioning idea
   - Look-up idea
   - Embedding idea

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   - Embedding idea
3. Further Work

Part I

What are nearest neighbors about?

Short overview of applications

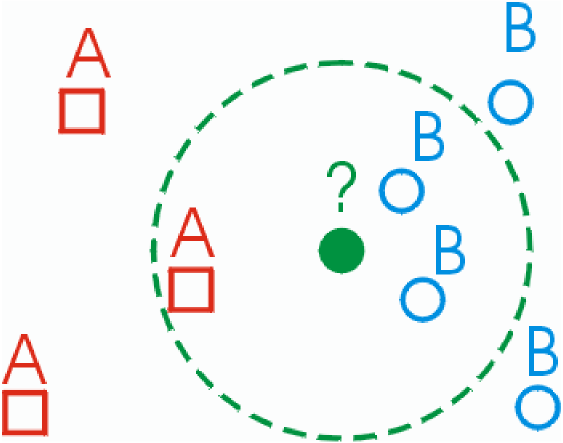
Variations of the problem

Informal Problem Statement

To preprocess a database of n objects
so that given a query object,
one can effectively determine
its nearest neighbors in database

First Application (1960s)

Nearest neighbors for classification:



Picture from http://cgm.cs.mcgill.ca/ soss/cs644/projects/perrier/Image25.gif

Applications in Web Technologies

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- Semantic search

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Formalization for nearest neighbors consists of:

- Representation format for objects
- Similarity function

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Remark 2: Accuracy of NN-based classification, prediction or recommendations depends solely on a data model, no matter what specific exact NN algorithm we use.

Variations of Data Model (1/3)

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  - Similarity: Hamming distance

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More data models?

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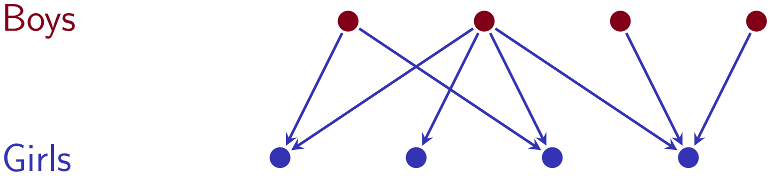

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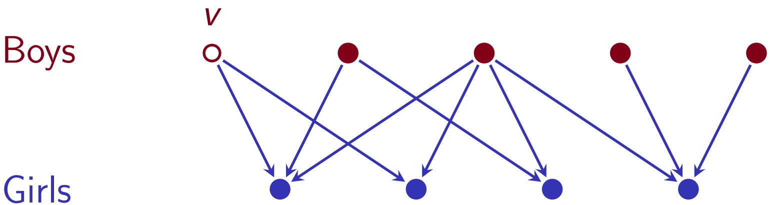

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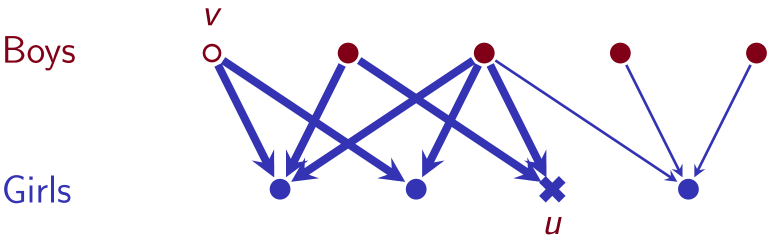

Variations of the Computation Task

- Approximate nearest neighbors
- Multiple nearest neighbors
- Nearest assignment
- All over-threshold neighbor pairs
- Nearest neighbors in dynamically changing database: moving objects, deletes/inserts, changing similarity function

Part II
Overview of Algorithmic Techniques

Partitioning, look-up and embedding-based approaches for **vector model**

New rare-point method (joint work with Hinrich Schütze)

Linear Scan

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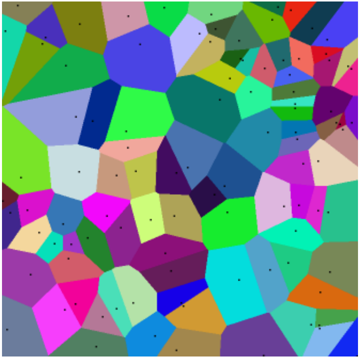
Directions for improvement:
order of scanning, pruning

Voronoi diagrams

Voronoi diagram is a mapping transforming every point p in database to the polygon of points for which p is the nearest neighbor



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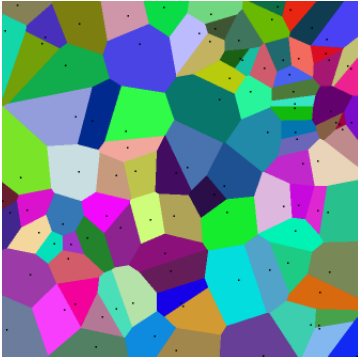

Can we generalize one-dimensional binary search?

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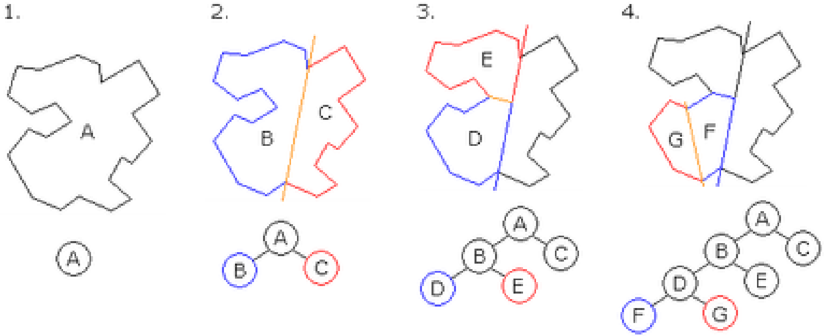
(Recursively) Go one step up, check whether the distance to the second branch is larger than that to current candidate neighbor if "yes" go up, else check this second branch

BSP-Trees

Generalization: BSP-tree allows to use any hyperplanes in tree construction



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Partitioning condition: $d(p, x) \ <? \ r$

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If $d(p, q) < r$ go to inner branch
If $d(p, q) > r$ go to outer branch and return minimum between obtained result and $d(p, q)$

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Retrieve all point that have at least one common nonzero component with the query point; Perform linear scan on them

Locality-Sensitive Hashing

Desired hash family $\mathcal{H}$:

- If $\|p-q\| \leq R$ then $\mathcal{P}r_{\mathcal{H}}[h(p)=h(q)] \geq p_1$
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Query processing:

Compute $A(q)$;
Find its nearest neighbor in R^l;

Part IV
Further Work

Directions for Applied Research

Directions for Theoretical Research

Questions to Practitioners

Directions for Applied Research

Attractive goals: **NN-based recommendation system, NN-based ads distribution system**

- Choose reasonable data model, add some assumptions about the nature of database and queries
- Find (theoretically) the best solutions in the resulting formalization
- Perform experimental analysis of obtained solutions
- Develop a prototype product

Directions for Theoretical Research

- Develop techniques for proving hardness of some computational problems with preprocessing. Find theoretical limits for some specific families of algorithms
- Extend classical NN algorithms to new data models and new task variations
- Develop theoretical analysis of existing heuristics. Average case complexity is particulary promising. Find subcases for which we can construct provably efficient solutions
- Compare NN-based approach with other methods for classification/recognition/prediction problems

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- Are you interested to apply NN approach in any of your future products?
- Give us benchmark data
- Give us names and contacts of potentially interested engineers

Summary

- Nearest neighbors is one of the key algorithmic problems for web technologies
- Key ideas: look-up tables, partitioning techniques, embeddings
- Further work: from algorithms to prototype products, from heuristics to theory, from canonical problem to new data models and new search tasks

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Thanks for your attention! Questions?

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